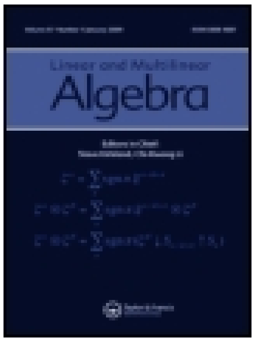

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On the unsolvability of inverse singular value problems almost everywhere

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ABSTRACT

In this paper, the unsolvability of inverse singular value problems almost everywhere is discussed. Applying the method described by Sun and Ye [J Comput Math. 1986;4:212–226], a sufficient condition is given such that the inverse singular value problems are unsolvable almost everywhere. This result shows that inverse singular value problems are unsolvable almost everywhere if the multiplicity of zero singular values is sufficiently large.

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1. Introduction

Throughout this paper, we shall use the following notational conventions. The symbol $\mathbb{R}^{m\times n}$ denotes the set of $m \times n$ real matrices, $\mathbb{R}^n = \mathbb{R}^{n\times 1}$. I_n is the $n \times n$ identity matrix. 0_n is the $n \times n$ null matrix. The superscript T is for transpose.

In this paper, we consider the following inverse singular value problem.

Problem ISVP: Given $p + 1$ real m–by–n matrices $B_0, B_1, \ldots, B_p$ with $m \geq n$, and q positive real numbers with an order $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_q > 0$, and $q + 1$ nonnegative integers $r_0, r_1, \ldots, r_q$ satisfying $r_0 + r_1 + \cdots + r_q = n$, find a real p-dimensional vector $\mathbf{c} = (c_1, c_2, \ldots, c_p)^T \in \mathbb{R}^p$ such that $B(\mathbf{c}) = B_0 + c_1B_1 + c_2B_2 + \cdots + c_pB_p$ has zero singular value of multiplicity r_0 and singular values $\sigma_1, \ldots, \sigma_q$ of multiplicity $r_1, \ldots, r_q$, respectively.

As a natural extension of inverse eigenvalue problems, the inverse singular value problem (ISVP) has a growing importance in practical applications. The special ISVP was originally proposed by Chu [1]. The ISVP arises in different applications such as the determination of mass distributions, orbital mechanics, irrigation theory, computed tomograghy, circuit theory, etc. [2–4]. Recently, some different ISVPs have been considered such as the low rank update of singular values [5] and the ISVP in some quadratic group [6]. Numerical methods for ISVP have been studied by several authors (see [1,7–11]). Chen [12] has derived an explicit expression of backward error for IVSP. For inverse

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eigenvalue problems of real symmetric matrices, Shapiro [13] gave a definition of the unsolvability almost everywhere and a sufficient condition such that it is unsolvable almost everywhere. Sun and Ye [14–16] have given some sufficient and necessary conditions such that inverse eigenvalue problems are unsolvable almost everywhere. Dai [17] discussed the unsolvability almost everywhere for inverse generalized eigenvalue problems. Their results show that the unsolvability of inverse (generalized) eigenvalue problems almost everywhere has a close relationship with multiplicity of eigenvalues.

The study of the unsolvability of the inverse singular value problem almost everywhere is important for the formulation of ISVP. Here, we will investigate the unsolvability almost everywhere for the ISVP. A sufficient condition is obtained such that the ISVP is unsolvability almost everywhere. Our result implies that the ISVP is unsolvable almost everywhere if the multiplicity of zero singular values is sufficiently large.

First, we extend the definition given by Sun and Ye [14] for unsolvability of the inverse eigenvalue problem almost everywhere to the ISVP.

Definition 1.1: The ISVP is said to be unsolvable almost everywhere if the set of matrices $B_0, B_1, \ldots, B_p \in \mathbb{R}^{m\times n}$ and vector $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_q)^T \in \mathbb{R}^q$ at which it is solvable has measure zero in the product vector space

$$\mathcal{K} = \underbrace{\mathbb{R}^{m\times n} \times \cdots \times \mathbb{R}^{m\times n}}_{p+1} \times \mathbb{R}^q. \tag{1.1}$$

The paper is organized as follows. In Section 2, we give a sufficient condition such that the ISVP is unsolvable almost everywhere and its proof. Concluding remarks are given in Section 3.

2. Main result

In this section, we will give a sufficient condition for the unsolvability of the inverse singular value problem almost everywhere. In order to prove our main result, the following lemmas are useful.

Lemma 2.1 [18]: *Let $\mathbf{f} = (f_1, f_2, \ldots, f_r)^T$ be a differentiable vector-value function defined in $\mathbb{R}^n$, and let $\mathcal{M} \subseteq \mathbb{R}^n$ be the set of points $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$ such that*

$$f_1(\mathbf{x}) = 0,\ f_2(\mathbf{x}) = 0,\ \ldots,\ f_r(\mathbf{x}) = 0.$$

Assume that for each point $\mathbf{x} \in \mathcal{M}$ the matrix

$$\frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_r}{\partial x_1} \\ \vdots & & \vdots \\ \frac{\partial f_1}{\partial x_n} & \cdots & \frac{\partial f_r}{\partial x_n} \end{pmatrix}$$

where each partial derivative is evaluated at $\mathbf{x}$ has rank r. Then $\mathcal{M}$ is an $(n-r)$-dimensional submanifold of $\mathbb{R}^n$.

Lemma 2.2 [14]: *Let $\mathcal{M}$ be an m-dimensional differentiable submanifold of an n-dimensional Euclidean space $\mathcal{E}$, $\mathcal{K}$ be a k-dimensional Euclidean space with $k < n$, and let* $\mathbf{F}$ *be a differentiable mapping of $\mathcal{M} \to \mathcal{K}$ defined by the following expressions:*

$$\begin{aligned} &y_1 = x_1, \ldots, y_k = x_k, \quad \textit{for} \quad \mathbf{x} = (x_1, \ldots, x_n)^T \in \mathcal{M}, \\ &\mathbf{F}(\mathbf{x}) = (y_1, \ldots, y_k)^T \in \mathcal{K}. \end{aligned}$$

If $m < k$, then $\mathbf{F}(\mathcal{M})$ is a set of measure zero in $\mathcal{K}$.

The following theorem shows that the ISVP is unsolvable almost everywhere if the inequality (2.1) is fulfilled.

Theorem 2.1: *If*

$$mr_0 - (n - r_0)(r_0 - 1) - p > 0, \tag{2.1}$$

then the ISVP is unsolvable almost everywhere.

Proof: We have three steps to prove Theorem 2.1.

(I) In terms of Lemma 2.2, define a differentiable mapping $\mathbf{F} : \mathcal{M} \longrightarrow \mathcal{K}$. Suppose that the IVSP is solvable at $B_0, B_1, \ldots, B_p \in \mathbb{R}^{m\times n}$ and $\boldsymbol{\sigma} = (\sigma_1, \ldots, \sigma_q)^T \in \mathbb{R}^q$. Then there exist a vector $\mathbf{c} = (c_1, \ldots, c_p)^T \in \mathbb{R}^p$, $U \in \mathbb{R}^{m\times n}$ and $V \in \mathbb{R}^{n\times n}$ such that

$$\begin{cases} B_0 + c_1B_1 + \cdots + c_pB_p = U\text{diag}(0_{r_0}, \sigma_1 I_{r_1}, \ldots, \sigma_q I_{r_q})V^T, \\ U^TU = I_n, \quad V^TV = I_n. \end{cases} \tag{2.2}$$

Let

$$U = (\mathbf{u}_1, \ldots, \mathbf{u}_n), \quad V = (\mathbf{v}_1, \ldots, \mathbf{v}_n)$$

be column partitions of U and V, respectively, and let

$$U_1 = (\mathbf{u}_{r_0+1}, \ldots, \mathbf{u}_n), \quad V_1 = (\mathbf{v}_{r_0+1}, \ldots, \mathbf{v}_n). \tag{2.3}$$

Then (2.2) can be written as follows

$$\begin{cases} B_0 + c_1B_1 + \cdots + c_pB_p = U_1\text{diag}(\sigma_1 I_{r_1}, \ldots, \sigma_q I_{r_q})V_1^T, \\ U_1^TU_1 = I_{n-r_0}, \quad V_1^TV_1 = I_{n-r_0}. \end{cases} \tag{2.4}$$

Furthermore we define the following Euclidean space:

$$\mathcal{E} = \underbrace{\mathbb{R}^{m\times n} \times \cdots \times \mathbb{R}^{m\times n}}_{p+1} \times \mathbb{R}^q \times \mathbb{R}^p \times \mathbb{R}^{m(n-r_0)} \times \mathbb{R}^{n(n-r_0)} \tag{2.5}$$

and

$$\mathcal{K} = \underbrace{\mathbb{R}^{m\times n} \times \cdots \times \mathbb{R}^{m\times n}}_{p+1} \times \mathbb{R}^q. \tag{2.6}$$

Let

$$\mathcal{M} = \left\{ \{B_0, B_1, \ldots, B_p, \boldsymbol{\sigma}, U_1, V_1\} \;\middle|\; \begin{array}{c} B_0, B_1, \ldots, B_p, \boldsymbol{\sigma}, U_1 \text{ and } V_1 \\ \text{satisfy the nonlinear system (2.4)} \end{array} \right\} \subset \mathcal{E}. \tag{2.7}$$

We define a differentiable mapping $\mathbf{F}$ of $\mathcal{M} \to \mathcal{K}$:

$$\mathbf{F}(X) = \{B_0, B_1, \ldots, B_p, \boldsymbol{\sigma}\} \in \mathcal{K}, \quad \text{for} \quad X = \{B_0, B_1, \ldots, B_p, \boldsymbol{\sigma}, \mathbf{c}, U_1, V_1\} \in \mathcal{M}.$$

(II) Prove that $\mathcal{M}$ is a $\left[pmn + p + q + (n - r_0)(m + r_0 - 1)\right]$-dimensional submanifold of $\mathcal{E}$.
Let $B_t = (b_{ij}^{(t)}),\ 0 \le t \le p$. Then we define the column vectors:

$$b_t = (b_{11}^{(t)}, \ldots, b_{1n}^{(t)}, b_{21}^{(t)}, \ldots, b_{2n}^{(t)}, \ldots; b_{m1}^{(t)}, \ldots, b_{mn}^{(t)})^T \in \mathbb{R}^{mn}, \quad 0 \le t \le p. \tag{2.8}$$

From (2.3) we also define the following column vectors:

$$\mathbf{u} = (\mathbf{u}_{r_0+1}^T, \ldots, \mathbf{u}_n^T)^T \in \mathbb{R}^{m(n-r_0)}, \quad \mathbf{v} = (\mathbf{v}_{r_0+1}^T, \ldots, \mathbf{v}_n^T)^T \in \mathbb{R}^{n(n-r_0)}. \tag{2.9}$$

Next we define differentiable real-valued functions $g_{ij}(1 \le i \le m, 1 \le j \le n)$, $h_{ij}(1 \le i,j \le n - r_0)$, and $l_{ij}(1 \le i,j \le n - r_0)$ in the Euclidean space $\mathcal{E}$ of (2.5) as follows:

$$\begin{aligned}
(g_{ij}) &= B_0 + \sum_{t=1}^{p} c_t B_t - (\mathbf{u}_{r_0+1}, \ldots, \mathbf{u}_n)\mathrm{diag}(\sigma_1 I_{r_1}, \ldots, \sigma_q I_{r_q})(\mathbf{v}_{r_0+1}, \ldots, \mathbf{v}_n)^T, \\
(h_{ij}) &= (\mathbf{u}_{r_0+1}, \ldots, \mathbf{u}_n)^T(\mathbf{u}_{r_0+1}, \ldots, \mathbf{u}_n) - I_{n-r_0}, \\
(l_{ij}) &= (\mathbf{v}_{r_0+1}, \ldots, \mathbf{v}_n)^T(\mathbf{v}_{r_0+1}, \ldots, \mathbf{v}_n) - I_{n-r_0}.
\end{aligned}$$

Noting that (h_{ij}) and (l_{ij}) are real symmetric matrix, we set

$$\mathbf{g} = (g_{11}, \ldots, g_{1n}, g_{21}, \ldots, g_{2n}, g_{m1}, \ldots, g_{mn})^T \in \mathbb{R}^{mn}, \tag{2.10}$$

$$\mathbf{h} = (h_{11}, \ldots, h_{1,n-r_0}, h_{22}, \ldots, h_{2,n-r_0}, \ldots, h_{n-r_0,n-r_0})^T \in \mathbb{R}^{\frac{(n-r_0)(n-r_0+1)}{2}}, \tag{2.11}$$

$$\mathbf{l} = (l_{11}, \ldots, l_{1,n-r_0}, l_{22}, \ldots, l_{2,n-r_0}, \ldots, l_{n-r_0,n-r_0})^T \in \mathbb{R}^{\frac{(n-r_0)(n-r_0+1)}{2}}, \tag{2.12}$$

and

$$\mathbf{f} = (\mathbf{g}^T,\ \mathbf{h}^T,\ \mathbf{l}^T)^T \in \mathbb{R}^{mn+(n-r_0)(n-r_0+1)}. \tag{2.13}$$

Each point $X = \{B_0, B_1, \ldots, B_p, \boldsymbol{\sigma}, \mathbf{c}, U_1, V_1\}$ in $\mathcal{E}$ is associated to a vector

$$\mathbf{x} = (\mathbf{b}_0^T, \mathbf{b}_1^T, \ldots, \mathbf{b}_p^T, \boldsymbol{\sigma}^T, \mathbf{c}^T, \mathbf{u}^T, \mathbf{v}^T)^T \in \mathbb{R}^{(p+1)mn+q+p+(m+n)(n-r_0)},$$

where $\mathbf{b}_t, \mathbf{u}, \mathbf{v}$ are represented by (2.8)–(2.9). Hence by the definition (2.7) of $\mathcal{M}$, we have $\mathbf{f}(\mathbf{x}) = 0$ for all $X \in \mathcal{M}$. Let $\mathbf{b} = (\mathbf{b}_0^T, \mathbf{b}_1^T, \ldots, \mathbf{b}_p^T)^T \in \mathbb{R}^{(p+1)mn}$. It is easy to verify from (2.10)–(2.13) that

$$\frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial \mathbf{g}}{\partial \mathbf{b}} & \frac{\partial \mathbf{h}}{\partial \mathbf{b}} & \frac{\partial \mathbf{l}}{\partial \mathbf{b}} \\ \frac{\partial \mathbf{g}}{\partial \boldsymbol{\sigma}} & \frac{\partial \mathbf{h}}{\partial \boldsymbol{\sigma}} & \frac{\partial \mathbf{l}}{\partial \boldsymbol{\sigma}} \\ \frac{\partial \mathbf{g}}{\partial \mathbf{c}} & \frac{\partial \mathbf{h}}{\partial \mathbf{c}} & \frac{\partial \mathbf{l}}{\partial \mathbf{c}} \\ \frac{\partial \mathbf{g}}{\partial \mathbf{u}} & \frac{\partial \mathbf{h}}{\partial \mathbf{u}} & \frac{\partial \mathbf{l}}{\partial \mathbf{u}} \\ \frac{\partial \mathbf{g}}{\partial \mathbf{v}} & \frac{\partial \mathbf{h}}{\partial \mathbf{v}} & \frac{\partial \mathbf{l}}{\partial \mathbf{v}} \end{pmatrix} = \begin{pmatrix} G & 0 & 0 \\ T_1 & 0 & 0 \\ T_2 & 0 & 0 \\ T_3 & H & 0 \\ T_4 & 0 & L \end{pmatrix},$$

where $T_i (i = 1, \ldots, 4)$ are some matrices of appropriate dimension, and

$$G = \frac{\partial \mathbf{g}}{\partial \mathbf{b}} = (I_{mn},\ c_1 I_{mn},\ \ldots,\ c_p I_{mn})^T \in \mathbb{R}^{(p+1)mn \times mn}, \tag{2.14}$$

$$H = \frac{\partial \mathbf{h}}{\partial \mathbf{u}}$$
$$= \left(\begin{array}{ccccc|ccccc|c|ccc|c} 2\mathbf{u}_{r_0+1} & \mathbf{u}_{r_0+2} & \cdots & \mathbf{u}_{n-1} & \mathbf{u}_n & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \mathbf{u}_{r_0+1} & \cdots & 0 & 0 & 2\mathbf{u}_{r_0+2} & \mathbf{u}_{r_0+3} & \cdots & \mathbf{u}_{n-1} & \mathbf{u}_n & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \mathbf{u}_{r_0+1} & 0 & 0 & 0 & \cdots & \mathbf{u}_{r_0+2} & 0 & \cdots & 2\mathbf{u}_{n-1} & \mathbf{u}_n & 0 \\ 0 & 0 & \cdots & 0 & \mathbf{u}_{r_0+1} & 0 & 0 & \cdots & 0 & \mathbf{u}_{r_0+2} & \cdots & 0 & \mathbf{u}_{n-1} & 2\mathbf{u}_n \end{array}\right)$$

and

$$L = \frac{\partial \mathbf{l}}{\partial \mathbf{v}}$$
$$= \left(\begin{array}{ccccc|ccccc|c|cc|c} 2\mathbf{v}_{r_0+1} & \mathbf{v}_{r_0+2} & \cdots & \mathbf{v}_{n-1} & \mathbf{v}_n & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & \mathbf{v}_{r_0+1} & \cdots & 0 & 0 & 2\mathbf{v}_{r_0+2} & \mathbf{v}_{r_0+3} & \cdots & \mathbf{v}_{n-1} & \mathbf{v}_n & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \mathbf{v}_{r_0+1} & 0 & 0 & 0 & \cdots & \mathbf{v}_{r_0+2} & 0 & \cdots & 2\mathbf{v}_{n-1} & \mathbf{v}_n & 0 \\ 0 & 0 & \cdots & 0 & \mathbf{v}_{r_0+1} & 0 & 0 & \cdots & 0 & \mathbf{v}_{r_0+2} & \cdots & 0 & \mathbf{v}_{n-1} & 2\mathbf{v}_n \end{array}\right).$$

Since

$$H^T H = L^T L = \mathrm{diag}(4, 2I_{n-r_0-1}, 4, 2I_{n-r_0-2}, \cdots, 4, 2, 4).$$

H and L are full column rank matrices. So we have

$$\mathrm{rank}(H) = \mathrm{rank}(L) = \frac{(n-r_0)(n-r_0+1)}{2}.$$

It is easy to see from (2.14) that

$$\mathrm{rank}(G) = mn.$$

Hence for each point $X \in \mathcal{M}$ the matrix $\frac{\partial \mathbf{f}}{\partial \mathbf{x}}$ in which each partial derivative is evaluated at X has

$$\begin{aligned}\operatorname{rank}\left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right) &= \operatorname{rank}(G) + \operatorname{rank}(H) + \operatorname{rank}(L) \\ &= mn + (n - r_0)(n - r_0 + 1).\end{aligned}$$

Hence by (2.13) we know that rank $\left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)$ is full column rank. Since

$$\dim(\mathcal{E}) = (p+1)mn + p + q + m(n - r_0) + n(n - r_0),$$

it follows from Lemma 2.1 that $\mathcal{M}$ is a submanifold of $\mathcal{E}$ with

$$\begin{aligned}\dim(\mathcal{M}) &= \dim(\mathcal{E}) - \operatorname{rank}\left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right) \\ &= pmn + p + q + (n - r_0)(m + r_0 - 1).\end{aligned}$$

(III) Prove that the ISVP is unsolvable almost everywhere. From the definition (2.6) of $\mathcal{K}$, we have

$$\dim(\mathcal{K}) = (p+1)mn + q.$$

Let

$$\mathcal{M}' = \mathbf{F}(\mathcal{M}).$$

Then it is easy to verify that if

$$mr_0 - (n - r_0)(r_0 - 1) - p > 0,$$

then $\dim(\mathcal{M}) < \dim(\mathcal{K})$. By Lemma 2.2 the set $\mathcal{M}'$ has measure zero under the assumption (2.1). Let $\mathcal{L}$ denote the set of points $X^{(1)} = \{B_0, B_1, \ldots, B_p, \boldsymbol{\sigma}\} \in \mathcal{K}$ at which the ISVP is solvable. Observe that for any point $X^{(1)} = \{B_0, B_1, \ldots, B_p, \boldsymbol{\sigma}\} \in \mathcal{L}$ there exist $\mathbf{c} \in \mathbb{R}^p$, $U_1 \in \mathbb{R}^{m\times(n-r_0)}$ and $V_1 \in \mathbb{R}^{n\times(n-r_0)}$ such that the point $X = \{B_0, B_1, \ldots, B_p, \boldsymbol{\sigma}, U_1, V_1\} \in \mathcal{M}$. Hence $\mathcal{L} \subset \mathcal{M}'$, and thus the set $\mathcal{L}$ has measure zero in the space $\mathcal{K}$. This means that if the condition (2.1) holds, the IVSP is unsolvable almost everywhere.

□

Remark 2.1: Theorem 2.1 shows that if the multiplicity of zero singular values is sufficiently large, the ISVP is unsolvable almost everywhere. In particular, when $m = n = p$, the inequality (2.1) reduces to $r_0(r_0 - 1) > 0$, which implies that the multiplicity of zero singular values is more than 1. In addition, if $r_0 = 0$ and $n > p$ or $r_0 = 1$ and $m > p$, then the IVSP is unsolvable almost everywhere. Hence, in a general way, it is impossible to solve the ISVP in these cases, unless these problems are treated in other sense, for example, in the sense of least square approximation (see [19]).

Remark 2.2: In [14–16], the authors gave some necessary conditions for the unsolvability of standard inverse eigenvalue problems almost everywhere. Similarly, what are necessary conditions for the unsolvability of the ISVP almost everywhere? This is an open problem.

3. Concluding remarks

In this paper, we give a sufficient condition such that the inverse singular value problem is unsolvable almost everywhere. Our result shows that if the multiplicity of zero singular values is sufficiently large, the ISVP is unsolvable almost everywhere. An interesting topic is to give numerical methods for the case of multiple zero singular values, which needs further investigation.

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