

2016 Taiwan Selection Test for PMWC and EMIC Preliminary Round Paper I (Time Allowed : 60 Minutes)

- Write down all answers on the answer sheet.
- Each problem is worth 5 points and the total is 100 points.

1. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 25 \times 125 = ?$
 $= (2 \times 5) \times (2 \times 2 \times 25) \times (2 \times 2 \times 2 \times 125) \times 2 = 10 \times 100 \times 1000 \times 2 = 2000000$
2. $5 + 10 + 15 + 20 + 25 + \dots + 300 + 305 + 310 + 315 = ?$
 $= \frac{(5 + 315) \times 63}{2} = 10080$
3. $10.08 + 20.16 + 30.32 + 40.64 + 51.28 - 8.07 - 12.15 - 16.31 - 20.63 - 25.27 = ?$
 $= (10 + 20 + 30 + 40 + 51 - 8 - 12 - 16 - 20 - 25)$
 $+ (0.08 - 0.07 + 0.16 - 0.15 + 0.32 - 0.31 + 0.64 - 0.63 + 0.28 - 0.27)$
 $= 70 + 0.05$
 $= 70.05$
4. $167 \times 399 + 167 \times 399 + 167 \times 399 + 167 \times 399 + 167 \times 399 + 167 \times 399 = ?$
 $= 167 \times 399 \times 6 = 167 \times 6 \times 399 = 1002 \times 399 = 399798$
5. $164 + 164 \times 3 + 164 \times 5 + 164 \times 7 + 164 \times 9 + 164 \times 11 = ?$
 $= 164 \times (1 + 3 + 5 + 7 + 9 + 11) = 164 \times 36 = (100 + 64) \times (100 - 64)$
 $= 100^2 - 64^2 = 10000 - 4096 = 5904$
6. $135789 + 357891 + 578913 + 789135 + 891357 + 913578 = ?$
 $= (1 + 3 + 5 + 7 + 8 + 9) \times 111111 = 33 \times 111111 = 3666663$
7. $201.6 \times 7.8 \div 9 \times 3 \div 67.2 \times 5 \div 3.9 = ?$
 $= 67.2 \times 3 \times 7.8 \div 9 \times 3 \div 67.2 \times 5 \div 3.9$
 $= 67.2 \div 67.2 \times 3 \times 3 \div 9 \times 7.8 \div 3.9 \times 5 = 2 \times 5 = 10$
8. $2016 + 2015 + 2014 + \dots + 1010 + 1009 - 1008 - 1007 - \dots - 3 - 2 - 1 = ?$
【Solution 1】
 $= (2016 - 1008) + (2015 - 1007) + (2014 - 1006) + \dots + (1010 - 2) + (1009 - 1)$
 $= \underbrace{1008 + 1008 + \dots + 1008 + 1008}_{1008 \text{ terms}}$
 $= 1008 \times 1008 = 1016064$
【Solution 2】
 $= (2016 + 2015 + 2014 + \dots + 2 + 1) - 2 \times (1008 + 1007 + \dots + 2 + 1)$
 $= \frac{(2016 + 1) \times 2016}{2} - 2 \times \frac{(1008 + 1) \times 1008}{2}$
 $= 1008 \times (2017 - 1009)$
 $= 1008 \times 1008 = 1016064$

$$\begin{aligned}
9. \quad & 954 \times 954 - 504 \times 504 - 450 \times 450 = ? \\
& = (954 + 504) \times (954 - 504) - 450 \times 450 \\
& = 1458 \times 450 - 450 \times 450 \\
& = (1458 - 450) \times 450 \\
& = 1008 \times 450 = 453600
\end{aligned}$$

$$\begin{aligned}
10. \quad & 2016 \times 7777 + 672 \times 6666 = ? \\
& = 2016 \times 7777 + 672 \times 3 \times 2222 \\
& = 2016 \times 7777 + 2016 \times 2222 \\
& = 2016 \times (7777 + 2222) \\
& = 2016 \times (10000 - 1) \\
& = 20160000 - 2016 \\
& = 20157984
\end{aligned}$$

$$\begin{aligned}
11. \quad & \frac{1 \times 3 \times 6 \times 10 + 4 \times 12 \times 24 \times 40 + 7 \times 21 \times 42 \times 70 + 10 \times 30 \times 60 \times 100}{2 \times 5 \times 7 \times 11 + 8 \times 20 \times 28 \times 44 + 14 \times 35 \times 49 \times 77 + 20 \times 50 \times 70 \times 110} = ? \\
& = \frac{1 \times 3 \times 6 \times 10 \times (1 + 4 + 7 + 10)}{2 \times 5 \times 7 \times 11 \times (1 + 4 + 7 + 10)} = \frac{1 \times 3 \times 6 \times 10}{2 \times 5 \times 7 \times 11} = \frac{18}{77}
\end{aligned}$$

$$\begin{aligned}
12. \quad & 20.16 \times 20.16 + 79.84 \times 120.16 = ? \\
& = 20.16 \times 20.16 + 79.84 \times (20.16 + 100) \\
& = 20.16 \times 20.16 + 79.84 \times 20.16 + 7984 \\
& = 20.16 \times (20.16 + 79.84) + 7984 \\
& = 20.16 \times 100 + 7984 \\
& = 2016 + 7984 = 10000
\end{aligned}$$

$$\begin{aligned}
13. \quad & 28.8 \times 199.99 - 2.88 \times 1999.6 - 0.288 \times 1.9993 = ? \\
& = 288 \times (0.1 \times 199.99 - 0.01 \times 1999.6 - 0.001 \times 1.9993) \\
& = 288 \times (19.999 - 19.996 - 0.0019993) \\
& = 288 \times 0.0010007 = 0.2882016
\end{aligned}$$

$$\begin{aligned}
14. \quad & 21 \frac{1}{16} + (5 \times 2 \frac{1}{7} - 4 \div \frac{1}{3}) \div \frac{1}{7} + 0.9375 = ? \\
& = 21 \frac{1}{16} + (5 \times \frac{15}{7} - 12) \div \frac{1}{7} + \frac{15}{16} = 22 - (\frac{84}{7} - \frac{75}{7}) \times 7 = 22 - \frac{9}{7} \times 7 = 13
\end{aligned}$$

$$\begin{aligned}
15. \quad & \frac{1169}{1690} \times 1 \frac{214}{501} \div 8 \frac{89}{114} = ? \\
& = \frac{1169}{1690} \times \frac{715}{501} \div \frac{1001}{114} \\
& = \frac{1169}{1690} \times \frac{715}{501} \times \frac{114}{1001} = \frac{7 \times 167}{2 \times 5 \times 13 \times 13} \times \frac{5 \times 11 \times 13}{3 \times 167} \times \frac{2 \times 3 \times 19}{7 \times 11 \times 13} = \frac{19}{13 \times 13} = \frac{19}{169}
\end{aligned}$$

$$16. 2015 \times \frac{2015}{2016} + 2016 \times \frac{2016}{2017} - \frac{4033}{2016 \times 2017} = ?$$

$$= 2015 \times \left(1 - \frac{1}{2016}\right) + 2016 \times \left(1 - \frac{1}{2017}\right) - \left(\frac{2016 + 2017}{2016 \times 2017}\right)$$

$$= 2015 - \frac{2015}{2016} + 2016 - \frac{2016}{2017} - \frac{1}{2017} - \frac{1}{2016}$$

$$= 2015 + 2016 - 2 = 4029$$

$$17. 91 + 929 + 9399 + 94999 + 959999 + 9699999 + 97999999 + 989999999 = ?$$

$$= (90 + 2 - 1) + (900 + 30 - 1) + (9000 + 400 - 1) + (90000 + 5000 - 1)$$

$$+ (900000 + 60000 - 1) + (9000000 + 700000 - 1) + (90000000 + 8000000 - 1)$$

$$+ (900000000 + 90000000 - 1)$$

$$= 90 + 900 + 9000 + 90000 + 900000 + 9000000 + 90000000 + 900000000$$

$$+ 2 + 30 + 400 + 5000 + 60000 + 700000 + 80000000 + 900000000 - 8$$

$$= 999999990 + 98765432 - 8$$

$$= 1000000000 - 10 + 98765432 - 8$$

$$= 1098765432 - 18$$

$$= 1098765414$$

$$18. \frac{1}{1 \times 2} - \frac{2}{1 \times 2 \times 4} + \frac{14}{1 \times 2 \times 4 \times 8} - \frac{30}{1 \times 2 \times 4 \times 8 \times 16} + \frac{62}{1 \times 2 \times 4 \times 8 \times 16 \times 32} = ?$$

$$= \frac{4}{1 \times 2 \times 4} - \frac{2}{1 \times 2 \times 4} + \frac{14}{1 \times 2 \times 4 \times 8} - \frac{30}{1 \times 2 \times 4 \times 8 \times 16} + \frac{62}{1 \times 2 \times 4 \times 8 \times 16 \times 32}$$

$$= \frac{2}{1 \times 2 \times 4} - \frac{14}{1 \times 2 \times 4 \times 8} + \frac{30}{1 \times 2 \times 4 \times 8 \times 16} - \frac{62}{1 \times 2 \times 4 \times 8 \times 16 \times 32}$$

$$= \frac{16}{1 \times 2 \times 4 \times 8} - \frac{14}{1 \times 2 \times 4 \times 8} + \frac{30}{1 \times 2 \times 4 \times 8 \times 16} - \frac{62}{1 \times 2 \times 4 \times 8 \times 16 \times 32}$$

$$= \frac{2}{1 \times 2 \times 4 \times 8} - \frac{30}{1 \times 2 \times 4 \times 8 \times 16} + \frac{62}{1 \times 2 \times 4 \times 8 \times 16 \times 32}$$

$$= \frac{32}{1 \times 2 \times 4 \times 8 \times 16} - \frac{30}{1 \times 2 \times 4 \times 8 \times 16} + \frac{62}{1 \times 2 \times 4 \times 8 \times 16 \times 32}$$

$$= \frac{2}{1 \times 2 \times 4 \times 8 \times 16} - \frac{62}{1 \times 2 \times 4 \times 8 \times 16 \times 32}$$

$$= \frac{64}{1 \times 2 \times 4 \times 8 \times 16 \times 32} - \frac{62}{1 \times 2 \times 4 \times 8 \times 16 \times 32}$$

$$= \frac{2}{1 \times 2 \times 4 \times 8 \times 16 \times 32}$$

$$= \frac{1}{2^{14}} = \frac{1}{16384} = 2^{-14}$$

$$\begin{aligned}
 19. & \left(20 \frac{485359}{999999} - \frac{323}{999}\right) \div 10 \frac{81018}{999999} = ? \\
 & = \left(\frac{20485339}{999999} - \frac{323323}{999999}\right) \div \frac{10081008}{999999} \\
 & = \frac{20162016}{999999} \times \frac{999999}{10081008} \\
 & = \frac{20162016}{10081008} \\
 & = 2
 \end{aligned}$$

$$20. \frac{33 \times 27 - 22 \times 18 + 9 - 4}{125 + \frac{(33 \times 27 - 22 \times 18 + 9 - 4)}{9 - \frac{(33 \times 27 - 22 \times 18 + 9 - 4)}{75 + \frac{(33 \times 27 - 22 \times 18 + 9 - 4)}{30 - \frac{(33 \times 27 - 22 \times 18 + 9 - 4)}{48 + \frac{(33 \times 27 - 22 \times 18 + 9 - 4)}{250}}}}} = ?$$

$$\begin{aligned}
 \text{Since } 33 \times 27 - 22 \times 18 + 9 - 4 &= (30 + 3) \times (30 - 3) + 9 - (20 + 2) \times (20 - 2) - 4 \\
 &= 30^2 - 3^2 + 9 - 20^2 + 2^2 - 4 \\
 &= 30^2 - 20^2 \\
 &= 500
 \end{aligned}$$

it is equivalent to

$$\begin{aligned}
 & \frac{500}{125 + \frac{500}{9 - \frac{500}{75 + \frac{500}{30 - \frac{500}{48 + \frac{500}{250}}}}} = \frac{500}{125 + \frac{500}{9 - \frac{500}{75 + \frac{500}{30 - \frac{500}{48 + 2}}}}} = \frac{500}{125 + \frac{500}{9 - \frac{500}{75 + \frac{500}{30 - \frac{500}{50}}}}} \\
 & = \frac{500}{125 + \frac{500}{9 - \frac{500}{75 + \frac{500}{30 - 10}}} = \frac{500}{125 + \frac{500}{9 - \frac{500}{75 + \frac{500}{20}}}} \\
 & = \frac{500}{125 + \frac{500}{9 - \frac{500}{75 + 25}}} = \frac{500}{125 + \frac{500}{9 - \frac{500}{100}}} = \frac{500}{125 + \frac{500}{9 - 5}} \\
 & = \frac{500}{125 + \frac{500}{4}} = \frac{500}{125 + 125} = \frac{500}{250} = 2
 \end{aligned}$$