2016 Taiwan Selection Test for IWYMIC
Preliminary Round (Time Allowed : 2 hours)

Section A: Questions requiring answers only. Each question is worth 5 marks.

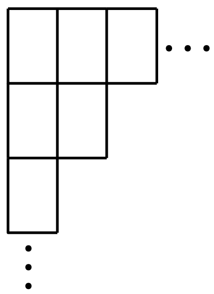
1. What is the remainder when 2^{2016} is divided by 13?

Ans : ________________

2. Class A has $2m$ boys and 13 girls while class B has 7 boys and $2n$ girls, where m and n are positive integers. Each student pays the same positive integral number of dollars into a fund, and the total amount of money raised by each class is $2mn+7m+13n+84$ dollars. What is the number of dollars paid by each student?

Ans : ____________ dollars

3. In a deck of 52 cards, each is 6 cm by 4 cm. Some of them are put together, without overlap, to form the largest possible square. Two adjacent cards must share a complete side of equal length. What is the number of cards left unused?



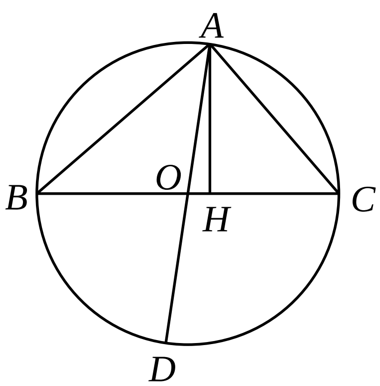
Ans : ____________ cards

4. How many different positive divisors does the following number have?

$$100^2 - 99^2 + 98^2 - 97^2 + \cdots + 42^2 - 41^2$$

Ans : ____________ divisors

5. B and C are points on a circle O with diameter AD, and on opposite sides of AD. H is the point on BC such that AH is perpendicular to BC. If $AH = 32$, $BH = 16\sqrt{5}$, $CH = 2\sqrt{185}$, what is the value of $AD \times AH$?



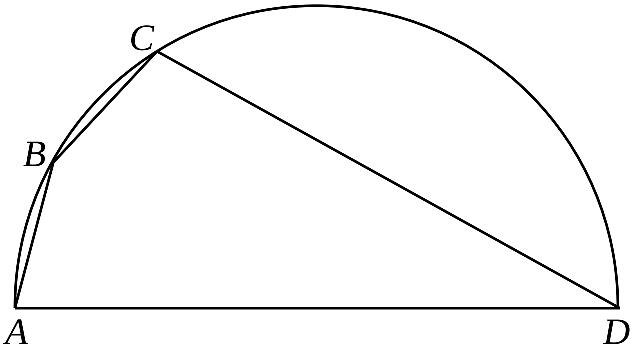
Ans : ________________

6. The positive integers are arranged in zig-zag fashion starting from the top left corner, as shown in the diagram below. The first four numbers in the diagonal from the top left are 1, 3, 7 and 13. What is the first number on this diagonal which is greater than 50?

1	2	9	10	$\cdots$
4	3	8	11	
5	6	7	12	
16	15	14	13	$\cdots$
$\vdots$			$\vdots$	

Ans : ________________

7. B and C are points on a semicircle with diameter AD, and B lies on the arc AC. If AD=4 cm and AB=BC=1 cm, what is the length of CD, in cm?

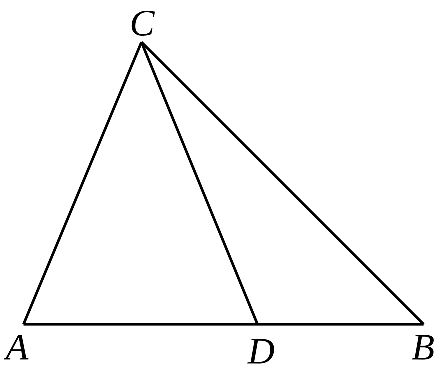


Ans : ________________ cm

8. What is the value of the positive number a if the difference between the two solutions of the equation $x^2+ax+1=0$ is 2?

Ans : ________________

9. D is a point on the side AB of triangle ABC such that AD=6 cm and $\angle ACD = 2\angle DCB = \angle B = 45^\circ$. What is the length of BD, in cm?

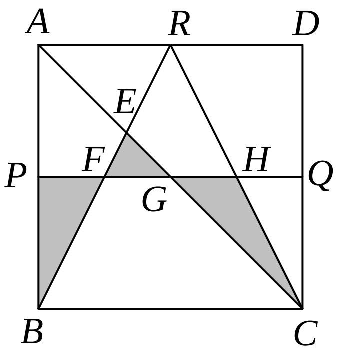


Ans : ________________ cm

10. Let a_1, a_2, a_3, $\cdots$, be real numbers such that for every positive integer n, $a_1 + 2a_2 + 3a_3 + \cdots + na_n = (n+1)^3$. What is the value of the expression $\dfrac{1}{a_1 - 1} + \dfrac{1}{2a_2 - 1} + \cdots + \dfrac{1}{49a_{49} - 1}$?

Ans : ________________

11. P, Q and R are the respective midpoints of the sides AB, CD and DA of a square $ABCD$. The segment BR intersects AC and PQ at E and F respectively, and the segment PQ intersects AC and RC at G and H respectively. If the total area of triangles BFP, EFG and CGH is m and the area of $ABCD$ is n, what is the value of $\frac{m}{n}$?



Ans : ________________

12. In a row of counters, each is either red or blue, and there is at least one of each color. Two counters with exactly 6 or 9 other counters in between must be of the same color. What is the maximum number of counters in this row?

Ans : ________ counters

Section B: Problems requiring full solutions. Each problem is worth 20 marks.

1. Let a and b be the legs of a right triangle and c the hypotenuse, where $a \neq b$. Let x and y be real numbers such that $\frac{x}{2a^2}+\frac{y}{c^2}=1$ and $\frac{x}{c^2}+\frac{y}{2b^2}=1$, prove that $x+y=2c^2$ 。

2. For any positive integer n, let $f(n)$ denote the sum of its digits. For example, $f(23) = 2 + 3 = 5$. How many positive integers n are there such that $\frac{n}{f(n)} > 8$?

3. Each of the numbers from 1 to 36 is placed in a different square of a 6 by 6 table. Consecutive numbers must be placed in squares sharing a common side. Prove that the sum of the 6 numbers on one of the diagonals is at most 174, and find a placement for which this maximum value is attained.
