DYNAMICS AND MODEL-BASED CONTROL FOR THE MOBILE MODULAR MANIPULATOR

YANGMIN LI AND YUGANG LIU

Department of Electromechanical Engineering, University of Macau, Taipa, Macao SAR, P.R.China,

Email: ymli@umac.mo, ya27401@umac.mo

Phone: (853) 3974464, Fax: (853)838314

A mobile modular manipulator which is composed of a three-wheeled mobile platform and a four-degree of freedom modular manipulator is investigated. The dynamics model of the entire mobile modular manipulator has been established using Lagrange formulation. An adaptive model-based controller is designed, and control simulations are performed to command the robotic end-effector to follow a desired trajectory. The simulation results demonstrate that the dynamics model and control method are valid to resolve the control issues of the mobile modular manipulators with nonholonomic constraints.

1. Introduction

A mobile modular manipulator is a kind of robot combining a modular manipulator with a mobile platform together as shown in Fig.1. A mobile manipulator with intelligence is an active research topic in recent years since it has many applications such as in modern factory for transporting materials, in dangerous situation for dismantling bombs or moving nuclear infected objects etc. Traditionally, a modular manipulator is mounted on a fixed base whose mobility is constrained. To extend the moving range of the manipulator, a mobile platform is attached to the modular manipulator in this paper in order to increase the workspace of the entire robot. However, building up the dynamics model for a mobile modular manipulator is a challenging task due to the interactive motions between the modular manipulator and the mobile platform. Also a trajectory following task becomes complex and difficult to realize. On the related research work, the dynamic characteristics between the mobile platform and the manipulator were studied. A path planning method was proposed for a mobile manipulator to execute multiple missions. A kind of motion control method
Figure 1. A mobile modular manipulator

with external force was presented. A kind of inverse kinematics solution for reconfigurable robots was presented. Parameters identification and vibration control have been investigated in our previous work. In this work, the dynamics model for the mobile modular manipulator is built and an adaptive model-based controller is designed for resolving the trajectory following problem.

This paper is organized as follows. The dynamics modeling of the mobile modular manipulator is presented in Section 2. An adaptive model-based controller is designed in Section 3. Simulations are made in Section 4. Section 5 gives some conclusions.

2. Dynamics Modeling

2.1. Kinematics Analysis

Without loss of generality, we analyze the three-wheeled mobile platform, which has two fixed wheels and a castor wheel. Two actuators are mounted on the shafts of the fixed wheels respectively to realize different rotation for the vehicle, see Fig. 2. Assume that the robot just move on a horizontal plane, then the transformation matrix of the 1st module of the modular manipulator with respect to the inertial base frame can be derived:

\[
B^0T = \begin{bmatrix}
C_m & -S_m & 0 & x_m + l_G \cdot C_m \\
S_m & C_m & 0 & y_m + l_G \cdot S_m \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (1)

Where \(C_m = \cos(\phi_m)\), \(S_m = \sin(\phi_m)\).

According to Denavit-Hartenberg notation, transformation matrix of
the $i$th module with respect to the inertial base frame can be derived by

$$ B_i^T = B_0^T \cdot Q_1^T \cdots \cdot i^{-1} T $$

(2)

Where $i^{-1} T$ is the transformation matrix of module $i$ with respect to module $i - 1$.

From the transformation matrix, we can obtain the position and posture vectors of the end-effector with respect to the inertial base frame. Furthermore, the posture can be determined by three independent parameters expressed by Z-Y-Z Euler angles, which are easy to measure and convenient for control. The relationship between Z-Y-Z Euler angles and posture vectors is:

$$
\begin{align*}
\phi &= \arctan 2(a_y, a_x) \\
\psi &= \arctan 2(a_z, -n_z) \\
\theta &= \arctan 2(a_x \cdot C_\phi + a_y \cdot S_\phi, a_z)
\end{align*}
$$

(3)

On the assumption of low speeds, the wheels of the mobile platform have no slip on forward, reverse, and sideways. The velocity constraints for the mobile modular manipulator include two parts as follows:

On the wheel plane

$$
\begin{bmatrix}
C_m & S_m & -d_m/2 \\
C_m & S_m & d_m/2 \\
C_{rm} & S_{rm} & -l_r \cdot S_r
\end{bmatrix}
\begin{bmatrix}
\dot{x}_m \\
\dot{y}_m \\
\dot{\phi}_m
\end{bmatrix}
- \begin{bmatrix}
0 & 0 & r_f \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\phi}_L \\
\dot{\phi}_R \\
\dot{\beta}_r
\end{bmatrix}
= 0
$$

(4)

Orthogonal to the wheel plane

$$
\begin{bmatrix}
-S_m & C_m & 0 \\
S_m & -C_m & 0 \\
-S_{rm} & C_{rm} & d_r - l_r \cdot C_r
\end{bmatrix}
\begin{bmatrix}
\dot{x}_m \\
\dot{y}_m \\
\dot{\phi}_m
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\cdot \dot{d}_r
= 0
$$

(5)
Where \( S_{rm} = \sin (\beta_r + \phi_m) \), \( C_{rm} = \cos (\beta_r + \phi_m) \), \( S_r = \sin (\beta_r) \), \( C_r = \cos (\beta_r) \)

Define \( \xi = [x_m \ y_m \ \phi_m \ \phi_L \ \phi_R]^T \), from Eqs. (4-5), we can easily obtain

\[
\dot{\xi} = \begin{bmatrix}
  r_f \cdot C_m / 2 & r_f \cdot C_m / 2 & r_f \cdot S_m / 2 & r_f / d_m \\
  r_f \cdot S_m / 2 & r_f \cdot S_m / 2 & r_f / d_m & -r_f / d_m \\
  -r_f / d_m & r_f / d_m & \frac{r_f}{2d_m} (d_m C_r + 2l_r S_r) & \frac{r_f}{2d_m} (d_m C_r - 2l_r S_r) \\
  \frac{r_f}{2d_m} (d_m S_r + 2d_r - 2l_r C_r) & \frac{r_f}{2d_m} (d_m S_r - 2d_r + 2l_r C_r) & 1 & 0 \\
  1 & 0 & 0 & 1
\end{bmatrix} \cdot \begin{bmatrix}
  \dot{\phi}_L \\
  \dot{\phi}_R \\
\end{bmatrix}
\]

(6)

Written as \( \dot{\xi} = S(\xi) \cdot [\dot{\phi}_L \ \dot{\phi}_R]^T \).

From Eqs. (4-5), we can also obtain

\[
A(\xi) \cdot \dot{\xi} = 0
\]

(7)

Written as \( A(\xi) \cdot \dot{\xi} = 0 \).

Substituting Eq. (6) into Eq. (7), we can obtain

\[
A(\xi) \cdot S(\xi) = 0_{5 \times 2}
\]

(8)

Linear and angular velocities of the end-effector can be calculated via the following equations:

\[
v_e = [\dot{p}_{ex} \ \dot{p}_{ey} \ \dot{p}_{ez}]^T, \omega_e = \dot{\phi}_m \cdot \vec{z} + \sum_{i=1}^{4} (\dot{q}_i \cdot B \cdot R \cdot \dot{z})
\]

(9)

Where \( \vec{z} = [0 \ 0 \ 1]^T \).

Expressed by Euler angles, the angular velocities can also be represented by

\[
\omega_e = \begin{bmatrix}
  0 -S\phi & C\phi S\theta & \dot{\phi} \\
  0 & C\phi S\theta & \dot{\theta} \\
  1 & 0 & C\theta
\end{bmatrix}
\]

(10)

Define \( q = [\phi_L \ \phi_R \ q_1 \ \cdots \ q_4]^T \) and \( x = [p_{ex} \ p_{ey} \ p_{ez} \ \phi \ \theta \ \psi]^T \), then according to Eqs. (3,9,10), we can obtain the differential kinematics:

\[
\dot{x} = J \cdot \dot{q}
\]

(11)
Where $J$ is the Jacobian matrix.

### 2.2. Dynamics Modeling

With an assumption of moving on a horizontal plane, the mobile platform has a constant potential energy $U_m$. Then, the Lagrange function can be calculated as follows

$$L = \frac{1}{2} \cdot m_c \cdot (\ddot{x}_m^2 + \ddot{y}_m^2) + \frac{1}{2} \cdot I_c \cdot \dot{\phi}_m^2 + \frac{1}{2} \cdot I_f \cdot \dot{\phi}_f^2 + \frac{1}{2} \cdot I_i \cdot \dot{\phi}_i^2 + \frac{1}{2} \cdot I_2 \cdot \dot{\phi}_2^2 - U_m + \sum_{i=0}^{4} \left( \frac{1}{2} \cdot m_i \cdot \dot{\vec{v}}_i^T \cdot \vec{v}_i + \frac{1}{2} \cdot \omega_i^T \cdot I_i \cdot \omega_i - m_i \cdot \dot{\vec{g}}^T \cdot \vec{p}_i \right)$$

(12)

Where $m_c$ is the mass of the cart including all driving units in the box; $I_c$ is their inertial moment of the cart; $I_f$ is the inertial moment of the front wheel, $I_1$ and $I_2$ are the inertial moments of the rear castor wheel related to its own axis and the vertical axis respectively; $\vec{g} = [0 \ 0 \ 9.81]^T$ is a gravity acceleration vector; $\vec{p}_i = [p_{ci}^x \ p_{ci}^y \ p_{ci}^z]^T$ is the coordinate vector of the mass center of the $i$th module with respect to frame $i$; $m_i$ and $I_i$ are the mass and the inertial moment for the $i$th module. $\vec{v}_i$ is the linear velocity of the center of mass for the $i$th module. $\omega_i$ is the angular velocity of the $i$th module with respect to frame $i$.

Define $\zeta = [\xi^T \ q_1 \cdots \ q_4]^T$, then the constrained dynamics for the non-holonomic mobile modular manipulator can be determined as follows:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\zeta}} \right)^T - \left( \frac{\partial L}{\partial \zeta} \right)^T = B(\zeta) \tau + C(\zeta) \lambda$$

(13)

Where $B(\zeta) = \begin{bmatrix} 0_{5 \times 6} \\ I_{6 \times 6} \end{bmatrix} \in \mathbb{R}^{11 \times 6}$, $\tau = [\tau_L \ \tau_R \ \tau_1 \cdots \ \tau_4]^T \in \mathbb{R}^6$, $C(\zeta) = \begin{bmatrix} A^T(\xi) \\ 0_{4 \times 5} \end{bmatrix} \in \mathbb{R}^{11 \times 5}$, $\lambda = [\lambda_1 \cdots \lambda_5]^T \in \mathbb{R}^5$; $\lambda_i = \lambda_i \left( \zeta, \dot{\zeta} \right)$ ($i = 1, 2, \cdots, 5$) are Lagrange multipliers associated with the constraints. $\tau_L, \tau_R, \tau_i$ are the driving torques of the left wheel, the right wheel and the $i$th joint respectively.

Separating the 1st and the 2nd order indexes from Eq. (13), we can obtain

$$H(\zeta) \cdot \dddot{\zeta} + V \left( \zeta, \dot{\zeta} \right) \cdot \dddot{\zeta} + G(\zeta) = B(\zeta) \cdot \tau + C(\zeta) \cdot \lambda$$

(14)

Where $H(\zeta) \in \mathbb{R}^{11 \times 11}$ is the inertial matrix, which is symmetric, positive definite; $V \left( \zeta, \dot{\zeta} \right) \in \mathbb{R}^{11 \times 11} \in \mathbb{R}^{11 \times 11}$ is the centripetal and coriolis matrix; $G(\zeta) \in \mathbb{R}^{11}$ denotes the gravitational forces.
From Eq. (6) and the definition of $\zeta$ and $q$, we can obtain

$$\dot{\zeta} = \begin{bmatrix} S(\zeta) & 0_{7\times4} \\ 0_{4\times2} & I_{4\times4} \end{bmatrix} \cdot \dot{q}$$

(15)

Written as $\dot{\zeta} = \bar{S}(\zeta) \cdot \dot{q}$.

Differentiating Eq. (15), we can obtain

$$\ddot{\zeta} = \dot{\bar{S}}(\zeta) \cdot \dot{q} + \bar{S}(\zeta) \cdot \ddot{q}$$

(16)

Substituting Eq. (16) into Eq. (14), and left multiplying $\bar{S}^T(\zeta)$, we can obtain

$$\bar{H}(\zeta) \cdot \ddot{x} + \bar{V}(\zeta, \dot{\zeta}) \cdot \dot{x} + \bar{G}(\zeta) = \bar{\tau}$$

(17)

Where $\bar{H}(\zeta) = \bar{S}^T(\zeta) \cdot H(\zeta) \cdot \bar{S}(\zeta)$, $\bar{V}(\zeta, \dot{\zeta}) = \bar{S}^T(\zeta) \cdot \bar{V}(\zeta, \dot{\zeta})$, $\bar{G}(\zeta) = \bar{S}^T(\zeta) \cdot \bar{G}(\zeta)$, $\bar{\tau}(\zeta) = \bar{S}^T(\zeta) \cdot B(\zeta) \cdot \tau$, and the term $\bar{S}(\zeta)^T \cdot C(\zeta) \cdot \lambda = 0_{6\times1}$ disappears.

According to Eq. (11), the dynamic equation can be transformed to Cartesian space

$$\tilde{H}(\zeta) \cdot \ddot{x} + \tilde{V}(\zeta, \dot{\zeta}) \cdot \dot{x} + \tilde{G}(\zeta) = \tilde{\tau}$$

(18)

Where $\tilde{H}(\zeta) = J^{-T} \cdot \bar{H}(\zeta) \cdot J^{-1}$, $\tilde{G}(\zeta) = J^{-T} \cdot \bar{G}(\zeta)$, $\tilde{\tau} = J^{-T} \cdot \bar{\tau}$, $\tilde{V}(\zeta, \dot{\zeta}) = J^{-T} \cdot [\tilde{V}(\dot{\zeta}) - \tilde{H}(\zeta) \cdot J^{-1} \cdot \tilde{J}] \cdot J^{-1}$.

**Properties:**

1) $\tilde{H}(\zeta)$ is positive definite and symmetric, i.e.

$$\tilde{H}^T(\zeta) = \tilde{H}(\zeta) > 0$$

(19)

2) Matrix $\tilde{N}(\zeta, \dot{\zeta}) = \tilde{H}(\zeta) - 2\tilde{V}(\zeta, \dot{\zeta})$ is skew-symmetric, i.e.

$$\tilde{N}^T(\zeta, \dot{\zeta}) = -\tilde{N}(\zeta, \dot{\zeta})$$

(20)

3) $\tilde{H}(\zeta), \tilde{V}(\zeta, \dot{\zeta}), \tilde{H}(\zeta)$ and $\tilde{\tau}$ are all bounded so long as the Jacobian matrix $J$ is nonsingular.

### 3. Model-based Controller Design

Assume the desired trajectory, velocity, and acceleration in the task space be $x_d$, $\dot{x}_d$, and $\ddot{x}_d$. The goal of model-based controller is to make $x$ approximate $x_d$ as close as possible.
Traditional model-based controller relies too much on precise dynamical parameters. However, in practice application, it is almost impossible to obtain accurate dynamic parameters for the mobile modular manipulator in advance. To suppress such errors as caused by parameter uncertainties, adaptive technique may be helpful. Assume that $\hat{\tilde{H}}(\zeta)$, $\hat{\tilde{V}}(\zeta, \dot{\zeta})$, and $\hat{\tilde{G}}(\zeta)$ represent the estimations of the inertial matrix, the Coriolis and centrifugal forces matrix, and the gravity forces matrix respectively. Then the following control scheme is designed, as shown in Fig. 3.

$$
\begin{align*}
\tau &= [\tilde{S}^T(\zeta) \cdot B(\zeta)^{-1} \cdot J^T \cdot \{\hat{H}(\zeta) \cdot \dot{x}_d + \tilde{V}(\zeta, \dot{\zeta}) \cdot \dot{x}_d + \hat{\tilde{G}}(\zeta) - K_P \cdot e\}] \\
\text{Where } K_P > 0 \text{ is the gain matrix for the proportional item.}
\end{align*}
$$

Combining Eq. (18) with Eq. (21), we can obtain the error equation

$$
\dot{\tilde{H}}(\zeta) \cdot \dot{e} + \dot{\tilde{V}}(\zeta, \dot{\zeta}) \cdot \dot{e} + K_P \cdot e = \hat{\tilde{H}}(\zeta) \cdot \dot{x}_d + \tilde{V}(\zeta, \dot{\zeta}) \cdot \dot{x}_d + \hat{\tilde{G}}(\zeta) \\
\text{Where } \hat{\tilde{H}}(\zeta) = \hat{H}(\zeta) - \tilde{H}(\zeta), \dot{\tilde{V}}(\zeta, \dot{\zeta}) = \hat{\tilde{V}}(\zeta, \dot{\zeta}) - \tilde{V}(\zeta, \dot{\zeta}), \tilde{G}(\zeta) = \hat{\tilde{G}}(\zeta) - \tilde{G}(\zeta)
$$

**Theorem 3.1.** If the Jacobian matrix $J$ keeps nonsingular, the system depicted in Eq. (22) is asymptotically stable under the following parameter adaptation law. The error signals are convergent along with time, i.e., $e(t)$
\[ \dot{e}(t) \to 0 \text{ as } t \to \infty. \]

\[
\begin{align*}
\dot{h}_{ij} &= -\Gamma_{Hij} \cdot \left[ e_i \cdot \ddot{x}_{dj} + \sigma_{Hij} \cdot |e_i| \cdot (\dot{h}_{ij} - \ddot{h}_{ij}) \right] \\
\dot{v}_{ij} &= -\Gamma_{Vij} \cdot \left[ e_i \cdot \ddot{x}_{dj} + \sigma_{Vij} \cdot |e_i| \cdot (\dot{v}_{ij} - \ddot{v}_{ij}) \right] \\
\dot{g}_i &= -\Gamma_{Gi} \cdot \left[ e_i + \sigma_{Gi} \cdot |e_i| \cdot (\dot{g}_i - \ddot{g}_i) \right]
\end{align*}
\]  

(23)

Where \( \Gamma_{Hij}, \Gamma_{Vij}, \Gamma_{Gi}, \sigma_{Hij}, \sigma_{Vij}, \) and \( \sigma_{Gi} \) are all positive constants.

**Proof:** Considering the nonnegative candidate Lyapunov function as follows

\[
V_S = \frac{1}{2} e^T \cdot \dot{H}(\zeta) \cdot \dot{e} + \frac{1}{2} e^T \cdot K_P \cdot e + \frac{1}{2} \sum_{j=1}^{6} \sum_{i=1}^{6} \left( \frac{\ddot{h}_{ij}^2}{\Gamma_{Hij}} + \frac{\ddot{v}_{ij}^2}{\Gamma_{Vij}} \right) + \frac{1}{2} \sum_{i=1}^{6} \left( \frac{\ddot{g}_i^2}{\Gamma_{Gi}} \right)
\]

(24)

The time derivative of Lyapunov candidate is

\[
\dot{V}_S = e^T \cdot \dot{H}(\zeta) \cdot \dot{e} + \frac{1}{2} e^T \cdot \dot{H}(\zeta) \cdot \dot{e} + e^T \cdot K_P \cdot \dot{e} + \sum_{j=1}^{6} \sum_{i=1}^{6} \left( \frac{\ddot{h}_{ij} \cdot \dddot{h}_{ij}}{\Gamma_{Hij}} \right) + \sum_{i=1}^{6} \left( \frac{\ddot{g}_i \cdot \dddot{g}_i}{\Gamma_{Gi}} \right)
\]

(25)

During the course of adaptive control, parameters \( \dot{h}_{ij}, \dot{v}_{ij}, \) and \( \dot{g}_i \) can be treated as constants, so

\[
\dot{h}_{ij} = \ddot{h}_{ij}, \dot{v}_{ij} = \ddot{v}_{ij}, \dot{g}_i = \ddot{g}_i
\]

(26)

Substituting Eq. (22) and Eq. (23) into Eq. (25), and considering the skew-symmetric property as shown in Eq. (20), yields

\[
\dot{V}_S = -\sum_{j=1}^{6} \sum_{i=1}^{6} \left( \sigma_{Hij} \cdot |e_i| \cdot \ddot{h}_{ij}^2 + \sigma_{Vij} \cdot |e_i| \cdot \ddot{v}_{ij}^2 \right) - \sum_{i=1}^{6} \left( \sigma_{Gi} \cdot |e_i| \cdot \ddot{g}_i^2 \right) \leq 0
\]

(27)

From Eq. (24) and Eq. (27), we can see that when and only when \( e = 0 \) and \( \dot{e} = 0 \), \( V_S \) and \( \dot{V}_S \) may equal to zeros. So, \( V_S \) is Lyapunov function. From LaSalle’s theorem, the system is asymptotically stable. If the Jacobian matrix keeps nonsingular, all the signals in the system are bounded. Then, we can conclude that \( e \to 0, \dot{e} \to 0 \) as \( t \to 0 \).

**4. Simulation Results**

In this simulation, the end-effector is commanded to follow the desired locus shown in Fig. 4. All the dynamic parameters are supposed to be
zeros at the beginning. Simulation time interval is selected as 5 seconds. The proportional gain matrixes is selected as $K_p = diag \{1.0\}$.

The constants used in the adaptive update law are chosen as follows

$$\Gamma_{Hij} = 0.1, \Gamma_{Vij} = 1.0, \Gamma_{Gi} = 1.0$$

$$\sigma_{Hij} = 0.05, \sigma_{Vij} = 0.001, \sigma_{Gi} = 0.1$$

The desired locus and the controlled one are both shown in Fig. 4. Figure 5 gives the tracking position errors and the Euler angle errors respectively. From these figures, we can see that small position and posture errors can be eliminated with this controller. If proper gain matrices are selected, the initial errors can be eliminated soon.

![Figure 4. A desired locus and the controlled result](image)

5. Conclusions

The kinematics and dynamics models of the entire mobile modular manipulator are established in consideration of nonholonomic constraints. An adaptive model-based controller is devised to control the end-effector to follow a desired trajectory, which can suppress the errors caused by parameter uncertainties effectively. Simulations are carried out to verify the effectiveness of the dynamics model and the adaptive model-based controller. This work is based on a combination of two robots: one three-wheeled mobile platform and a 4-DOF modular manipulator in our laboratory. However, the dynamics modeling method and the adaptive model-based controller
proposed in this paper can also be used to study other mobile manipulators. The future investigation will be performed on field experiments of the mobile modular manipulator.

References


