Dynamic Modeling and Optimization of a Decoupled XY Flexure Parallel Micro-Manipulator

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Abstract: The architecture optimization of a newly designed flexure XY parallel micro-manipulator with both input and output decoupling is conducted in this paper. In view of the compliance of flexure hinges, the input stiffness model of the motion stage are established based upon the matrix method, and then the dynamic equation is derived through the Lagrangian approach, which is verified by the modal analysis performed via finite element analysis (FEA). With the goal of maximizing the natural frequency of the stage, the architecture optimization is carried out by the particle swarm optimization (PSO) along with the stiffness values of actuators and workspace size of the manipulator taken into account. Both the effectiveness of the optimization and the decoupling properties of the manipulator are validated by the FEA with ANSYS. The results are useful for the development of a new decoupled XY flexure parallel micro-manipulator for micro/nano scale manipulation.

Keywords: Micro/Nano Manipulator, Flexure Mechanism, Parallel Robot, Decoupled Manipulator, Optimal Design.

1. INTRODUCTION

The compliant positioning mechanisms based on flexure hinges have been applied more and more widely in situations where a high precision motion within a micro range is required. Typical applications of such kind of mechanisms can be found in micro-manipulation fields, micro-assembly tasks, MEMS sensors and actuators and so on. For instance, an XY translational stage is needed in an atomic force microscope (AFM) for the implementation of scanning of the probe over the samples to get such information as surface profile of the scanned materials.

The flexures instead of conventional mechanical joints endow a mechanism with several advantages including no backlash, no friction, vacuum compatibility and easy to manufacture [1]. Besides, a parallel kinematic structure are widely adopted in a flexure mechanism due to its contribution to high rigidity, high load-carrying capacity, and high accuracy without cumulative errors. Many successful applications of flexure parallel kinematic manipulators have been reported in the literature.

However, most of the proposed positioning stages have a coupled motion [2]. In some situations where the sensory feedback of output displacements of the stage is not allowed, a decoupled stage with proper calibrations is preferred [3]. Generally, a decoupled stage means that one actuator produces only one axial direction motion without influences on the motion with respect to other axes. The term of “decoupled” refers to the output decoupling of the stage motion. In contrast, the input decoupling is seldom paid attention since it emphasizes on the isolation of the input motion instead. When the stage is driven by one motor, other motors may suffer from unwanted loads due to the motion of the output platform, or there may exist clearances between other motors and the interfacing points with the stage [4]. Corresponding to output decoupling, the input decoupling can be defined as the isolation of actuators for a compliant stage.

The design of a totally decoupled flexure parallel stage with both input and output decoupling is a challenging work even for an XY positioning stage. In previous works dedicating to flexure XY parallel stages, the design of a totally decoupled one is attempted in [3]. However, the mechanism possesses a complex architecture complicating the fabrication process of the stage.

In recent works of the authors, a totally decoupled flexure parallel XY manipulator is proposed as shown in Fig. 1. The objective of this paper is to present the architecture optimization of the XY stage with the consideration of kinetostatics and dynamics properties.
2. ARCHITECTURE DESCRIPTION OF THE XY STAGE

As elaborated in Fig. 1, the designed XY stage consists of four limbs and possesses a symmetry structure with piezoelectric actuation. The major advantage of PZT is the large blocking force whereas the main drawback lies in the small travel stroke with comparison to other types of linear actuators. Each limb of the manipulator consists of a compound parallelogram flexure that is connected to the actuator through a decoupler. The decoupled output motion of the stage is enabled by the adoption of compound parallelogram. Besides, thanks to a large transverse stiffness of the decoupler, the input decoupling of the actuator through a decoupler is to absorb the transverse load and transmit the actuation force of the actuator. The multifunction of the decoupler allows the generation of an XY stage with a simple structure with comparison to the existing ones.

In the following discussions, the stiffness modeling of the stage is derived based on the matrix method and then used in the dynamic modeling process by resorting to Lagrangian approach.

3. COMPLIANCE MODELING BASED ON MATRIX METHOD

The matrix-based compliance/stiffness modeling method [5, 6] employed for flexure mechanism design is revisited in this section.

3.1 Compliance of Flexure Hinge and Transformation

The compliance matrix of a right circular flexure hinge with the coordinate frame shown in Fig. 2 can be written as:

\[
C_h = \begin{bmatrix}
\frac{\Delta x}{\Delta x} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\Delta y}{\Delta y} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\Delta z}{\Delta z} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\Delta x}{\Delta x} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\Delta y}{\Delta y} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\Delta z}{\Delta z}
\end{bmatrix}
\]

where the compliance factors in the matrix have been derived in several references, and the equations with best accuracy as reviewed in [7] are adopted in this paper. The \( C_h \) represents the compliance of end point \( O_j \) with respect to the other end point which is fixed as the base.

Referring to Fig. 2, the local compliance \( C_i = C_h \) of a flexure hinge can be transferred into another coordinate \( O_j \) by:

\[
C_i' = T_i^j C_i (T_i^j)^T
\]

in which the transformation matrix can be written as:

\[
T_i^j = \begin{bmatrix}
R_i^j & S(r_i^j)R_i^j \\
0 & R_i^j
\end{bmatrix}
\]

where \( R_i^j \) is the rotation matrix of coordinate \( O_i \) with respect to \( O_j \), \( r_i^j \) is the position vector of point \( O_i \).

3.2 Compliance of Flexure Serial Chains

As for a kinematic chain shown in Fig. 3(a) which consists of \( k \) hinges connected in serial, the compliance \( C_P \) of the serial chain can be defined as the compliance of the free-end \( P \) with respect to the fixed-end \( O_0 \) of the linkage.

Intuitively, when a force \( F_P \) is applied at the end point \( P \) of the chain, the induced deflection \( X_P \) at that point is contributed by each flexure hinge involved in the serial chain. The displacement \( X_P \) can be obtained by the superposition of the deflection \( X_P^k \) of the end point due to the elastic deformation of the \( i \)-th hinge, i.e.,

\[
C_P F_P = X_P = \sum_{i=1}^{k} X_i^P
\]

\[
= \sum_{i=1}^{k} C_i^P F_P = \sum_{i=1}^{k} T_i^P C_i (T_i^P)^T F_P, \quad (5)
\]

which gives an expression for the compliance matrix at point \( P \):

\[
C_P = \sum_{i=1}^{k} T_i^P C_i (T_i^P)^T, \quad (6)
\]

where \( C_i \) is the compliance of the \( i \)-th hinge expressed in its local frame \( O_i \), which is transferred into frame \( P \) at the free-end by the transformation matrix \( T_i^P \).

It should be noted that the compliance \( C_P \) is derived in different ways as in [5] and [6]. The proposed derivation process is a more intuitive approach.

3.3 Compliance of Flexure Parallel Mechanisms

Let’s consider a flexure parallel mechanism (see Fig. 3(b)) consists of \( n \) (\( n \geq 2 \)) serial chains, which connect from the base \( O_0 \) to the mobile platform at point \( P_j \) in parallel. The compliance \( C_{P_j} \) of each chain is expressed at frame \( P_j \), for \( j = 1, 2, \ldots, n \).

The stiffness matrix for the flexure parallel mechanism expressed in frame \( B \) can be derived as:

\[
K_B = \sum_{j=1}^{n} (T_{P_j}^B)^{-T} K_{P_j} (T_{P_j}^B)^{-1}, \quad (7)
\]
where $T_{Pj}$ is the transformation matrix from frame $P_j$ to $B$, and $K_{Pj} = C_{Pj}^{-1}$ is the stiffness matrix of the $i$-th chain. Then, the compliance of the parallel mechanism can be obtained as: $C_B = K_B^{-1}$.

### 4. STIFFNESS MODELING OF THE XY STAGE

The stiffness of the XY stage can be defined with respect to either the output mobile platform or the input end of the stage. The output stiffness refers to the stiffness at point $B$ (where the external force $F_B$ is exerted) with respect to the ground. On the contrary, the input stiffness solves the stiffness at point $A$ (where the input force $F_A$ is applied) with respect to the ground. In this section, the input stiffness model is established based on the aforementioned matrix method.

#### 4.1 Input Stiffness Determination

##### 4.1.1 Output Compliance of One Displacement Amplifier

Due to the double symmetry of the amplifier, one quarter as shown in Fig. 4 is picked out for the purpose of analysis. The output compliance $\text{^4}C_D$ is defined as the compliance of point $D$ (where the external force $F_D$ is exerted) with respect to the input end $O_1$.

According to Eq. (6), the compliance of $D$ with respect to $O_2$ and $O_6$ can be respectively written as:

$$\text{^2}C_D = T_3^D C_3(T_3^P)^T + T_5^D C_5(T_5^P)^T,$$

$$\text{^6}C_D = T_7^D C_7(T_7^P)^T + T_9^D C_9(T_9^P)^T,$$

where the local compliances $C_i = C_h$ (for $i = 3, 5, 7, 9$), and the transformation matrices $T_i^D$ can be easily obtained.

In view of the parallel connection between the two chains $DO_1O_3$ and $DO_5O_6$, the compliance $\text{^4}C_D$ can be derived in accordance with Eq. (7):

$$\text{^4}C_D = (\text{^1}K_D)^{-1} = (\text{^2}K_D + \text{^6}K_D)^{-1}$$

$$= [(\text{^2}C_D)^{-1} + (\text{^6}C_D)^{-1}]^{-1}.$$  

Let $\text{^4}C_D$ denote the output compliance of top-left part of the amplifier with respect to the ground, then $\text{^4}C_D = \text{^4}C_D$. Due to the top-down symmetry of the amplifier, the

![Fig. 3 (a) A flexure serial chain with external force applied at the end point. (b) A flexure parallel mechanism with applied external force.](image)

![Fig. 4 Parameters and coordinates for one quarter of the displacement amplifier.](image)

![Fig. 5 Parameters and coordinates of one quarter of the XY stage.](image)
Furthermore, the overall compliance at point $P$ with respect to the ground can be determined as:

$$C_P = T_B^P C_D (T_D^B)^T + D C_P,$$

(15)

where $T_B^P$ is the transformation matrix from coordinate $D$ to $P$.

**4.1.3 Output Compliance of the XY Stage**

Once the amplifier is connect to the limb of the XY stage at point $D$ as a decoupler, it will tolerate the force applied by the remainder of the stage at the interfacing point $D$. Excluding the amplifier in limb 2, the stiffness model of the stage is graphically illustrated in Fig. 6. The stiffness model can be interpreted as that the three parallel limbs (1, 3, and 4) are connected at the center point $B$ and then connected to point $D$ through limb 2 whose compliances come from the two flexure serial chains between $B$ and $D$.

Similar to the calculation of $D C_P$, the stiffness $B K_D$ of point $D$ with respect to the mobile platform center $B$ can be derived as well. Then, the stiffness due to the three parallel limbs (no. 1, 3, 4) can be determined by:

$$L_{134} K_B = \sum_{i=1,3,4} (T_{P_i}^B)^{-T} (C_{P_i})^{-1} (T_{P_i}^B)^{-1},$$

(16)

where $C_{P_i} = C_P$ and $T_{P_i}^B$ is the corresponding transformation matrix.

Therefore, the compliance of the XY stage without the amplifier in limb 2 can be obtained as:

$$C_D = B C_D + T_B^P (L_{134} K_B)^{-1} (T_D^B)^T.$$  

(17)

**4.1.4 Input Stiffness of the XY Stage**

The free body diagram of one quarter of the amplifier is shown in Fig. 7, where the load $F_{Dy}$ denotes the force along the $y$ direction applied by the XY stage excluding the amplifier when the amplifier is actuated with the force $F_{in}$ created by the linear actuator.

With the assumption that the output end $D$ is fixed, it is not difficult to derive the input compliance $D^* C_A$ of one quarter of the amplifier based on the matrix method. Considering the force-deflection relationships at the input end, we can get the following equations:

$$u_{in} = c_{11} F_{in} + c_{12} F_{Dy} + c_{16} M_{Az},$$

(18)

$$u_{Ay} = c_{21} F_{in} + c_{22} F_{Dy} + c_{26} M_{Az},$$

(19)

$$0 = c_{61} F_{in} + c_{62} F_{Dy} + c_{66} M_{Az},$$

(20)

where $c_{ij}$ and $c_{6i}$ (for $i = 1, 2, 6$) are compliance factors in the $i$-th row of the matrix $D^* C_A$. The four unknowns $u_{in}, u_{Ay}, F_{Dy}$, and $M_{Az}$ can be derived in terms of $F_{in}$ in view of three relationships expressed by Eqs. (18) ~ (20) along with another equation, which can be obtained by taking into account the relationship between the load and deflection along the $y$ direction:

$$u_{Ay} = -d_{22} F_{Dy},$$

(21)

where $d_{22}$ is a compliance factor of the matrix $C_D$, and the negative sign indicates the opposite directions between the force $F_{Dy}$ and deflection $u_{Ay}$ at the input end $A$.

Then, a necessary calculation allows the derivation of the input compliance of the XY stage as follows.

$$C_{stage}^{in} = \frac{u_{in}}{F_{in}} = c_{11} = \frac{c_{16} c_{61}}{c_{66}} - \left( c_{12} - \frac{c_{16} c_{62}}{c_{66}} \right) \frac{c_{21} c_{66} - c_{26} c_{61}}{c_{22} c_{66} - c_{26} c_{62} + d_{22} c_{66}}.$$

(22)

Moreover, the output motion of the amplifier $u_{Ay}$ can be obtained as a function of the input displacement $u_{in}$:

$$u_{Ay} = A_u u_{in} = \frac{d_{22} (c_{21} c_{66} - c_{26} c_{61}) u_{in}}{c_{22} c_{66} - c_{26} c_{62} + d_{22} c_{66}} C_{stage}^{in},$$

(23)

where $A_u$ denotes the amplification ratio of the amplifier.

**4.2 Amplification Ratio Determination**

With reference to Fig. 6, we can see that when the stage is driven by the input motion $u_{in}$, the displacement of point $D$ along the $y$ direction is $u_{Dy} = u_{Ay}$ as derived in Eq. (23). In view of the identical value of reaction forces along the $y$ direction at positions $D$ and $B$, the output displacement at the mobile platform center can be determined as:

$$u_{By} = \frac{b_{22}}{d_{22}} u_{Dy},$$

(24)
where $b_{22}$ denotes a compliance factor of the compliance matrix $L_{134}C_B$.

Therefore, with the consideration of Eqs. (23) and (24), the amplification ratio of the XY stage can be calculated as:

$$A_s = \frac{u_{TB}}{u_{Tin}} = \frac{b_{22}(c_{21}c_{66} - c_{26}c_{61})}{(c_{26}c_{66} - c_{26}c_{62} + d_{22}c_{66})} C_{\text{stage}}^{\text{st}}. \quad (25)$$

**5. DYNAMIC MODELING OF THE XY STAGE**

### 5.1 Dynamics Analysis

In order to fully describe the free vibrations of the XY stage, the independent of the secondary stage associated with the compound parallelogram flexure in each limb should be considered. Thus, the generalized coordinate vector is selected as $q = [q_1 \ q_2 \ u_1 \ u_2 \ u_3 \ u_4]^T$, where the coordinates are shown in Fig. 1.

In order to express the kinetic and potential energies of the entire XY stage, the intermediate variables describing the output motion of the mobile platform ($x$, $y$), output displacements ($d_i$) and rotation angles ($\theta_i$, for $i = 1$ to 4) of links associated with the $i$-th amplifier are assigned, which can be expressed by the generalized coordinates only. Then, the kinetic and potential energies ($T$ and $V$) for the entire stage can be expressed in terms of the generalized coordinates only. Based on the Lagrange equation:

$$\frac{d}{dt} \frac{\partial T}{\partial q_i} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = F_i, \quad (26)$$

the dynamic equation describing a free motion of the XY stage can be derived as:

$$M \ddot{q} + K q = 0, \quad (27)$$

where the two $6 \times 6$ mass and stiffness matrices take on the following forms:

$$M = \begin{bmatrix} M_{11} & 0 & 0 & M_{14} & 0 & M_{16} \\
0 & M_{22} & M_{23} & 0 & M_{25} & 0 \\
0 & M_{32} & M_{33} & 0 & 0 & 0 \\
M_{41} & 0 & 0 & M_{44} & 0 & 0 \\
0 & M_{52} & 0 & 0 & M_{55} & 0 \\
M_{61} & 0 & 0 & 0 & 0 & M_{66} \end{bmatrix}, \quad (28)$$

$$K = \begin{bmatrix} K_{11} & 0 & 0 & 0 & 0 & 0 \\
0 & K_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & K_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & K_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & K_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & K_{66} \end{bmatrix}. \quad (29)$$

The expressions for the matrix factors are omitted here for the reason of brevity. Based on the theory of vibrations, the following modal equation describing free vibration of the system can be obtained:

$$(K - \lambda_j M) \Phi_j = 0, \quad (30)$$

for $j = 1$ to 6. The condition of non-zero solutions for Eq. (30) can be derived by

$$|K - \lambda_j M| = 0, \quad (31)$$

which allows the calculation of the eigenvalues, i.e., $\lambda_j = \omega_j^2$ with $\omega_j$ denoting the natural cyclic frequency of the system. Then, the natural frequency can be computed as $f_j = \frac{1}{2\pi}\omega_j$.

### 5.2 Modal Analysis with FEA

In order to verify the evaluated natural frequency of the XY stage, the modal analysis via FEA is undertaken with ANSYS. The parameters of the XY stage are described in Table 1, and the mesh model is created with the element PLANE 82. It is observed that the first natural frequency obtained by FEA is up to 101.4 Hz, while the frequency evaluated by the dynamics analysis is 111.9 Hz.

Taking the FEA results as the benchmark, one can observe that the derivation of the derived model from the FEA result is around 10%. The offset mainly comes from the accuracy of the adopted equations for the compliance factors and the neglect of the compliances of links between flexure hinges.

Furthermore, the simulation result of the third modal shape for the XY stage is illustrated in Fig. 8, which shows that once the four secondary stages suffer from external forces around the same direction, the mobile platform of the stage exhibits a large rotation in the $x$-$y$ plane. Hence, such kind of loads should be avoided in practical applications in order not to induce parasitic rotation of the output platform.

**6. ARCHITECTURE OPTIMIZATION OF THE XY STAGE**

In order to develop an XY stage, it is a key step to determine its dimensions by taking into account its performances simultaneously. To increase the natural frequency of the stage, the weight of the mobile platform is reduced by removing unnecessary masses. The stroke of the two PZT is assumed to be 20 $\mu$m.

### 6.1 Optimization Statement

With the selection of natural frequency of the stage as an objective function, the optimization is stated as follows:

- Maximize: Natural frequency ($f$)
- Variables to be optimized: $r$, $t$, $l_1$, $l_2$, and $l_8$
- Subject to:
  1. Amplification ratio $A_s \geq 6$
  2. Input stiffness value $K_{in} \leq K_{\text{PZT}}$

<table>
<thead>
<tr>
<th>Table 1 Main parameters of the XY stage.</th>
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<tr>
<td>Architectural parameters (mm)</td>
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<td>$l_4$</td>
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<td>24</td>
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<tr>
<td>Material parameters</td>
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<tr>
<td>Young’s modulus</td>
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<td>------------------</td>
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<tr>
<td>71.7 GPa</td>
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3. Parameter ranges: $2 \text{ mm} \leq r \leq 6 \text{ mm}$, $0.3 \text{ mm} \leq t \leq 2 \text{ mm}$, $5 \text{ mm} \leq l_1 \leq 20 \text{ mm}$, $1 \text{ mm} \leq l_2 \leq 5 \text{ mm}$, and $30 \text{ mm} \leq l_3 \leq 100 \text{ mm}$

As far as a material with a specific thickness is concerned, five parameters need to be optimized since other parameters can be expressed by considering the length and width of the PZT with the addition of a proper assembling space. In addition, the amplification ratio of the stage should not exceed the stiffness of the adopted PZT, i.e., $K_{PZT} = 10 \text{ N} / \mu\text{m}$. Besides, since the stage will be manufactured by the wire-EDM (electrical discharge machining) process, the thinnest portion of the notch hinge should be no less than 0.3 mm corresponding the maximum tolerance of ±0.01 mm. The upper bounds for design variables are all limited so as to generate a compact manipulator.

6.2 PSO Optimization and Results

It has been shown that particle swarm optimization (PSO) is superior to the genetic algorithm (GA) in terms of optimization performance [8]. Thus, the PSO is adopted in the current optimization, and the optimized stage dimensions are: $r = 2.0 \text{ mm}$, $t = 0.5 \text{ mm}$, $l_1 = 13.6 \text{ mm}$, $l_2 = 2.2 \text{ mm}$, and $l_3 = 45.0 \text{ mm}$. Additionally, the corresponding performances are: $A_s = 6.0$, $K_{in} = 10 \mu\text{m}$, and $f = 173.7 \text{ Hz}$.

The FEA is carried out to verify the performances of the optimized stage. The simulation results from ANSYS reveal that the XY stage has a workspace size of $116 \times 116 \mu\text{m}^2$ indicating an amplification ratio of 5.8. Besides, the input stiffness is $9.9 \text{ N} / \mu\text{m}$ and the natural frequency is up to 163.5 Hz, which exhibits the efficiency of the performed optimization. In addition, when the stage translates along the $x$ direction due to the actuation of PZT 1 with an input displacement $20 \mu\text{m}$, the cross-talk of the mobile platform in the $y$ direction is 0.03% and the rotation around the $z$-axis is 0.9 $\mu$rad, which are all neglectable and hence reveal a well-decoupled motion of the stage. Moreover, the maximal transverse displacement at output end of decoupler 2 is less than 0.5 $\mu\text{m}$, i.e., less than 0.5% of axial displacement of the decoupler, which indicates the well isolation property of actuators.

7. CONCLUSIONS

In this paper, the dimension optimization of a flexure XY parallel micro-manipulator with both input and output decoupling is implemented. The analytical models for input stiffness, amplification ratio, and natural frequency have been established and employed in the architecture optimization. The optimization is carried out by the PSO approach with the goal of maximizing the natural frequency along with the stiffness values of actuators and workspace size of the stage taken into account. Both the effectiveness of the optimization and the decoupling properties of the manipulator are validated by the FEA with ANSYS.

The presented results are helpful for the development of a new XY flexure parallel stage. In the future work, the performances of the stage will be evaluated by experimental studies, and the manipulator will be adopted in micro/nano scale manipulation.

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REFERENCES