Abstract—This paper presents the design and analysis of a novel compliant flexure-based micro-parallel positioning stage for micro active vibration isolation application. The stage is constructed with a symmetric structure by employing three parallel PUU legs, a moving platform and a fixed platform. It is driven by 6 electromagnetic actuators and with 3 translational DOFs. The mobility characters of the stage is analyzed and proved via FEA method. The compliance modeling of the stage is conducted by resorting to compliance matrix method, and analytical models for electromagnetic forces are also established, both mechanical structure and electromagnetic model are validated by finite element analysis (FEA) performed with ANSYS. The mechanical structure is analyzed in a multi-physics environmental simulation and electromagnetic actuators are applied in ANSYS too. Both FEA and the analytical models well demonstrate that the movement of the stage is purely translational. The prototype of the designed system is fabricated, preliminary test shows the design is successful. With the parameters designed in the paper, the stage can have large working space, very high resolution and heavy work-load ability as well.

I. INTRODUCTION

The parallel robotic manipulator attracts many researchers’ attentions and it has also increasing applications in robotics, machine tools, positioning devices, measurement systems, and so on. It has been proved that such a spatial closed loop manipulator has its great potential and advantages over the traditional serial manipulator. The parallel manipulator is able to provide a very complicated motion for the end-effector in a flexible way with many degrees of freedom (DOF) by using a combination of prismatic, revolute, universal joints, etc [1]. So, the family of parallel manipulators is very large [2].

There are many industrial applications which need a pure translational motion such as a motion simulator, a positioning tool in an assembly line, high-air level working stage for cleaning building windows etc [3]. In addition, many researchers are studying parallel manipulators for these applications recently due to their high stiffness and high load capacity. Tsai [4] studied kinematics of a UPU (Universal-Prismatic-Universal) parallel manipulator, and systemically enumerate the parallel manipulator [5]. Di Gregorio suggested a new 3-DOF translational parallel}

manipulator with 3-URC (Universal-Revolute-Cylindrical) leg configuration [6] and he also studied carefully about the kinematic condition of pure wrist parallel robot [7]. Most of these manipulators are realized in macro size such as famous Delta 3-DOF translational motion robot, which is used in industry for fast picking-up and putting down operation. Serial manipulators are also applied in performing automatic assembling job such as peg-in-hole task [8] in macro world, but in micro world some micromanipulators or stages are required to do similar works.

In this paper, with a purpose of constructing a 3-DOF translational micro/nano positioning stage, some flexure hinge-based micro parallel structures and electromagnetic actuators are adopted. A flexure machine with notched hinges has been widely used for micro-precision machinery [9]. The advantages of the flexure hinge-based mechanism for micro/nano manipulation are vacuum compatibility, no backlash property, no non-linear friction, simple structure and easy manufacture. Flexure parallel mechanism (FPM) possesses high flexibility and high natural frequencies. There is no error accumulation and no need for lubrication. Because of these advantages, the FPM is widely employed as the ultra-precision manipulation system in various fields such as fiber optic coupling, micro-machining, vibration isolation device, and semiconductor production [10]-[12]. Many micro motion drivers such as piezoceramics (PZT) and shape memory alloy (SMA) can be selected as the actuators. Although the PZT can provide a very high-precision driving output, it can only behave a very small working range. As for the SMA, it has a large moving range, but has time delay response and heating/cooling problems. For both large moving range, high-resolution and large output force, electromagnetic actuators are selected as the drivers for 3-DOF translational flexure-based micro parallel manipulator for active vibration isolation application. With comparison to the PZT directly actuated micro manipulator, it owns a larger workspace. Compared with the maglev micro manipulator, it possesses a high payload capability. Besides, the proposed electromagnetic driving 3-DOF translational micro positioning stage can be fabricated easily with very low cost.

Based on our previous work [13], the conceptual design is conducted and the compliance of the stage is analyzed via matrix method and validated by FEA method in ANSYS. The prototype of the stage is fabricated, preliminary experiments show that the mobility feature of the stage is mainly translational and the working range is quite large.

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II. ARCHITECTURE DESCRIPTION OF THE MANIPULATOR

The 3-PUU translational micro parallel manipulator is shown in Fig.1, which consists of a mobile plate, a fixed base, and three limbs with identical kinematic structure. Each limb connects the fixed base to the mobile plate by a \( P \) (prismatic) joint and two \( U \) (universal) joints in sequence, where \( P \) joint is the active joint driven by a linear actuator assembled on the fixed base. Thus, the mobile platform is attached to the base by three identical \( PUU \) linkages.

To facilitate the analysis, as shown in Fig.1 and Fig.2, we assign a fixed Cartesian frame \( O\{x, y, z\} \) at the centered point \( O \) of the fixed base, and a moving Cartesian frame \( P\{u, v, w\} \) on the mobile platform at the centered point \( P \), along with the \( z \)- and \( w \)-axes perpendicular to the platform, and the \( x \)- and \( y \)-axes parallel to the \( u \)- and \( v \)-axes, respectively.

The \( i \)-th leg \((i = 1, 2, 3)\) of the 3-PUU mechanism is shown in Fig.2, in which the \( s_{ij}, (i = 1, 2, 3, j = 1, 2, 3, 4) \) is the unit vector of the axis of the \( j \)-th revolute pair of the \( i \)-th leg and \( s_{i0} \) is the unit vector of \( O\bar{A} \). Point \( A_i \) is the intersection of the two revolute pairs of the \( U \) joint that connects the \( i \)-th leg to the \( P \) joint before it is fixed to the base. Point \( B_i \) is the center point of the axes of the two revolute pairs of the universal joint that connects the \( i \)-th leg to the mobile plate. Finally the length \( d_i \) is the distance of \( A_iB_i \), and \( d_1 = d_2 = d_3 = d \), since the three legs are identical.

In order to obtain pure translational motion characteristics of the platform with respect to the base, with these notations, the mobile plate and the base must meet the following geometric condition, which was deeply discussed in [3]-[4].

\[
s_{11} \cdot s_{21} = s_{13} \cdot s_{23}; s_{11} \cdot s_{31} = s_{13} \cdot s_{33}; s_{21} \cdot s_{31} = s_{23} \cdot s_{33},
\]

(1)

Where \( s_{i2} = s_{i4}, s_{i1} = s_{i3}(i = 1, 2, 3) \), for simplicity.

We assemble parallel mechanism as \( s_{11} \perp s_{i6}, s_{i1} \perp s_{i2}, s_{i1} = s_{i3}(s_{i1}/s_{i3}), s_{i2} = s_{i4}(s_{i2}/s_{i4}) \) and \( \triangle B_1B_2B_3 \) is equilateral triangle and \( \triangle A_1A_2A_3 \) is equilateral triangle at the initial time, that will fulfill the requirement mentioned above.

\[C_{1}^{O} = T_{t}^{O}C_{1}(T_{t}^{O})^{T}\]

(2)

A. Compliant Modeling of Flexure Mechanisms Based on Matrix Method

Various methods can be used to model the compliant mechanisms [14]. Among these typical methods, compliance matrix method, which has been well developed in architecture to analyze a rigid structure like bridges, is widely adopted into analyzing compliant mechanisms [14]-[15].

Generally, a flexure-hinge mechanism consists of \( n \) individual flexure elements connected in serial or parallel manners. In order to obtain the compliance model of the entire mechanism, the local compliances need to be transformed to a common(global) frame chosen to describe the mechanism. Then, compliances connected in serial and stiffness connected in parallel can be added together to generate the entire one.

B. Compliance Modeling of the P-joint

The compliance \( C_{1}^{O} \) is defined as the compliance of point \( O_1 \) with respect to the ground point \( O \) as shown in Fig.3 and Fig.4. Additionally, as shown in Fig.3, the local compliance of the hinge is defined as \( C_{ij}(6 \times 6) \) [4]. All the limbs and hinges are connected in serial or parallel. In order to calculate the compliance of the stage, compliance of every flexural hinge with respect to the ground \( C_{1}^{O} \) can be derived by

Considering the symmetrical structure of the P-joint, each limb \( L_{O_1O_3}, L_{O_2O_4}, L_{O_5O_7} \) and \( L_{O_6O_8} \) is a serial chain with two flexural hinges. The local compliance matrices of each flexural hinge are \( C \), for limb \( O_1O_3 \), we have \( r_{AO_1} = [0, 0, -l_2/2] \) and \( r_{AO_3} = [0, l, -l_2/2] \), the transformation matrix is
Fig. 4. Parameters and structure of a flexure P joint.

Fig. 5. Structure of a flexure based U joint.

The compliance matrix of the limb $O_1O_3$ with respect to the point $A$ can be denoted as $C_{L_1O_1O_3}^A$ and derived by

$$C_{L_1O_1O_3}^A = T_{O_1}^A C'[T_{O_3}^A]^T + T_{O_1}^A C'[T_{O_3}^A]^T$$

(3)

where $C' = T_2(\pi/2)C[T_2(\pi/2)]^T$, and $T_2(\pi/2)$ is the rotation matrix around the local $x$-axis on the flexure as shown in Fig.3 [2].

Just the same as obtaining of $C_{L_1O_1O_3}^A$, we can get the compliance matrix of $C_{L_2O_2O_4}^B$, $C_{L_3O_3O_5}^B$. Then the stiffness of the P-joint with respect to the ground point $O$ described as $K_{P_{joint}}^O = [C_{P_{joint}}^O]^{-1}$ can be derived by

$$K_{P_{joint}}^O = \{T_{O_1}^A[(C_{L_1O_1O_3}^A)^{-1} + (C_{L_2O_2O_4}^B)^{-1}]^{-1}[T_{O_3}^A]^T\}^{-1}$$

(4)

$$+ \{T_{O_1}^A[(C_{L_3O_3O_5}^B)^{-1} + (C_{L_2O_2O_4}^B)^{-1}]^{-1}[T_{O_3}^A]^T\}^{-1}$$

$$C_{P_{joint}}^O = [K_{P_{joint}}^O]^{-1}$$

(5)

C. Compliance Modeling of the U-joint and the leg

As shown in Fig.5, an U joint can be treated as two parallel R joints, and the two R joints just rotate $\pi/2$ with respect to $x$-axis, the local compliance matrix of an U joint can be derived by

$$C_U = C + T_x(\pi/2)C[T_x(\pi/2)]^T$$

(6)

where $T_x(\pi/2)$ is the rotation matrix around the local $x$-axis on the flexure as shown in Fig.3 [2]. The local compliance of the leg can be derived by

$$C_L = C_U + T_{A'}^B C_U [T_{A'}^B]^T$$

(7)

As shown in Fig.6, $r_{A'B'} = [d, 0, 0]$ for $T_{A'}^B$. Then the compliance matrix of the leg with respect to the ground reference point can be described as that the leg rotates around $y'$-axis first, then rotates around $x$-axis and moves translational along $y'$-axis to connect with the P-joint.

$$C_{L'}^O =$$

$$T_{A'}^O T_{y'}(\beta_i) T_x(\gamma_i) T_z(\pi/2) C_L [T_x(\pi/2)]^T [T_z(\gamma_i)]^T [T_y(\beta_i)]^T$$

(8)

where $T_x(\gamma_i)$, $T_y(\beta_i)$ and $T_z(\pi/2)$ is the rotation matrix around the $x$-axis, $y$-axis and $z$-axis as shown in Fig.2 with angles of $\gamma_i$, $\beta_i$ and $\pi/2$.

For each PUU leg, we can have the whole compliance matrix as follows:

$$C_{Limb-i} = C_{P_{i}}^O + C_{P_{joint-i}}^O$$

(9)
D. Compliance Modeling of the Whole Micromanipulator

The whole micromanipulator can be treated as three symmetric legs connected in parallel. The output compliance matrix $C^O_P$ can be defined as the compliance from point on the top center of the mobile stage $P$ to ground point $O$. It can be derived by

$$K^O_P = [C_{Limb-1}]^{-1} + [T_z(2\pi/3)C_{Limb-2}[T_z(2\pi/3)]^T]^{-1} + [T_z(4\pi/3)C_{Limb-3}[T_z(4\pi/3)]^T]^{-1}$$  \hspace{1cm} (10)$$

and

$$C^O_P = [K^O_P]^{-1}$$  \hspace{1cm} (11)$$

The input stiffness can be defined as the stiffness from point $A_i$ to $O$, denoted as $C^O_{A_i}$, which can be obtained as the same process as the calculation of the output stiffness.

III. Compliance Model Validation with FEA

The established models for the assessment of input and output compliance of the micro-parallel Positioning stage are validated by the FEA through ANSYS software. The architecture parameters of the stage are described in Table I where all the hinges are designed as identical dimensions, and the mesh model is created with the element SOLID185. The FEA model of P joint is built up and simulated in the ANSYS software. When a force is applied at the input face of the P joint, the corresponding output motion of the P joint can be observed in Fig.7. To simplify the analysis, the mechanical model for FEA analysis in ANSYS software is establish as an integrated one. The FEA model of the stage can be observed in Fig.8, the relationship between the input force and the output motion displacement can be obtained to determine the input and output compliance. The stage performances evaluated as the derived models and FEA are elaborated in Table II. Taking FEA results as the benchmark, we can observe that the maximum deviation of the derived model from the FEA results is around 20 percent. The offset mainly comes from the accuracy of the adopted equations for the compliance factors and the neglect of compliances of the links between flexure hinges since these links are assumed to be rigid in the matrix model. When the FEA model in ANSYS is driven by exerting forces on the $A_i$ points, the moving stage moves totally in translational along $x$-, $y$-, $z$-direction. Given arbitrary input, the responses are shown in Fig.9. It shows clearly that the mobile platform is mainly translational motion with respect to its initial position. Furthermore, the modal analysis is conducted in FEA model via ANSYS. The results of natural frequencies are listed in Table III. It can be observed in Fig.10 that the first mode is mainly up and down translational movement, the second mode mainly is horizontal movement, the third mode is...
mainly rotational movement, and the fourth mode is wrap of the end-plate movement. The mode frequencies of the first two mode shapes are 67.5Hz and 135Hz respectively and are far lower than the rotation and wrap one. That is to say, when the stage is driven, the motion is mainly translational, even if a very high dynamic driven signal or disturbance may cause some rotations and wrap movement.

IV. PRELIMINARY EXPERIMENTAL STUDY

After the manipulator is carefully modeled and analyzed, the parameters are carefully selected. The 3-dimensional prototype is designed and virtually assembled as shown in Fig.11. The structural parameters are the same as the mechanical model for FEA analysis in ANSYS software. The 3D prototype shows clearly that the stage is with 3-PUU structure. On each P-joint, there are two electrodynamic actuators (voice-coil actuators). One electromagnetic actuator is fixed on the center of mobile beam of the P-joint, another one is fixed on the fixed stage. The advantages of the design with two electromagnetic actuators are to enlarge the driving force and the working range compared with the design of only one electromagnetic actuator along with iron armature plate. The parameters of the electromagnetic actuators such as the numbers of the coil turns, the diameters of the coil wire, the diameter and the length of the coil, the area of the armatures, all can be selected properly according to the compliance performance of the mechanical system as studied before [16]-[17]. The power amplifier for the electromagnetic actuators is made through computational analog amplifier OPA548, which is a low-cost, high-voltage/high-current operational amplifier with high resolution. It operates from either single(+8V to +60V) or dual supplies(4V to 30V) and outputs high current 3A continuously and 5A at peak. The setup of the whole experimental system is shown in Fig.12.

After all the mechanical systems are designed and parameters are carefully selected, the mechanical parts are made through electro-discharge machining. High resolution contactless capacitive sensors are assembled appropriately to measure the motion of mobile stage.

Since all the mechanical system and the electro amplifier driving system are fabricated and assembled carefully, preliminary experiments are launched to test the performance of the design. The first experiments are to test the electromagnetic actuators and the P-joint as shown in Fig.14. In this simple test, it can be seen that the central mobile beam on the
| Architectural parameters (mm) | \( r | t | w | d | |OA_i| |O_B_i| | l_1 | l_2 |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 0.5 | 14 | 135 | 30 | 101 | 40 | 35 |

**TABLE II**
KINETOSTATICS PERFORMANCE OF A GRIPPER

<table>
<thead>
<tr>
<th>Performance</th>
<th>Output Stiffness(N/\mu m)</th>
<th>Input Stiffness(N/\mu m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix model</td>
<td>0.22</td>
<td>0.38</td>
</tr>
<tr>
<td>FEA</td>
<td>0.19</td>
<td>0.32</td>
</tr>
<tr>
<td>Deviation(%)</td>
<td>15.8</td>
<td>17.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model sequence</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>FEA (Hz)</td>
<td>67.5</td>
<td>135</td>
<td>310.5</td>
<td>319.5</td>
</tr>
</tbody>
</table>

P-joint moved clearly and the moving range can reach more than 1mm. Definitely, this double electromagnetic actuators design can realize a large working range for the top mobile stage as preliminary design.

After the motion of P-joint along with the electromagnetic actuators is tested and the 3-PUU legs are assembled, the experiment about the motion of the mobile stage is launched. Since the motion of the mobile stage can be observed through eyes clearly when the P-joints are driven by the electromagnetic actuators as shown in Fig.14, the motion range of the mobile stage can reach a very large size of working range as shown in Fig.15. As studied in section II, the input compliance of the mechanical system is \( 0.38 N/\mu m \). Aiming for generating a movement range of \( \pm 500 \mu m \) of the mobile stage, at least \( \pm 190 N \) driving force is needed. The force that the electromagnet selected in this research can generate 348N, which is enough for driving the stage. Moreover, it can be observed form Fig.15 that the motion of the mobile stage is mainly translational and no rotational motion is observed. The previous motion characteristic analysis can be simply validated here via this experiment.

It is well known that electromagnetic actuators possess hysteresis characteristics naturally, which bring an accuracy issue on controlling the designed 3-PUU micromanipulator. Just as carefully studied in our previous work [18]-[19], the similar model and control method will be used in this research to deal with the hysteresis problems in the designed 3-PUU micromanipulator.

**V. CONCLUSION**

A conceptual design and fabrication of a novel flexure-based 3-DOF micro-positioning stage driven by electromagnetic actuators are proposed in this paper. The stage possesses a kind of translational mobility characteristics, simple symmetrical structure, and large motion range. The compliance matrix-based method is applied to model the mechanical structure. The mechanical system model is validated by FEA method via ANSYS software. Preliminary experiments are launched, the motion range of the manipulator is quite large. Since both the capacitive displacement sensors and an accelerometer are installed in the prototype measurement system, the designed manipulator can be used as a high accuracy 3-DOF micro/nano positioning stage for bio-engineering or micro-assembly application, which can also be used as a micro-parallel active vibration isolation system. In our future work, closed-loop control strategy will be adopted to control the designed manipulator under the real-time control environment in MATLAB Simulink and dSPACE system.

**REFERENCES**