A Totally Decoupled Piezo-Driven XYZ Flexure Parallel Micropositioning Stage for Micro/Nanomanipulation

Yangmin Li, Senior Member, IEEE, and Qingsong Xu, Member, IEEE

Abstract—This paper reports the design and development processes of a totally decoupled flexure-based XYZ parallel-kinematics micropositioning stage with piezoelectric actuation. The uniqueness of the proposed XYZ stage lies in that it possesses both input and output decoupling properties with integrated displacement amplifiers. The input decoupling is realized by actuation isolation using double compound parallelogram flexures with large transverse stiffness, and the output decoupling is implemented by employing two-dimensional (2-D) compound parallelogram flexures. By simplifying each flexure hinge as a two-degree-of-freedom (2-DOF) compliant joint, analytical models of kinematics, statics, and dynamics of the XYZ stage are established and then validated with finite-element analysis (FEA). The derived models are further adopted for optimal design of the stage through particle swarm optimization (PSO), and a prototype of XYZ stage is fabricated for performance tests. The nonsymmetric hysteresis behavior of the piezo-stage is identified with the modified Prandtl-Ishlinskii (MPI) model, and a control scheme combining the inverse model-based feedforward with feedback control is constructed to compensate the plant nonlinearity and uncertainty. Experimental results reveal that a submicron accuracy 1-D and 3-D positioning can be achieved by the system, which confirms the effectiveness of the proposed mechanism and controller design as well.

Note to Practitioners—Motivated by the requirement of developing a decoupled XYZ micropositioning stage for 3-D micro/nanomanipulation uses, a novel spatial parallel mechanism incorporating flexure hinges is presented in this paper, and piezoelectric actuators (PZTs) owning large output force and stiffness are used for actuation. The piezo-stage has the merits of not complicated structure as well as both input and output decoupling properties. By input decoupling, the PZTs are isolated and protected. With output decoupling, the parallel stage behaviors like a serial one, which enables the adoption of single-input-single-output (SISO) controller for each axis. Before the fabrication of the stage, its parameters are optimized to achieve a high resonant frequency under performance constraints in terms of workspace size, input stiffness, and safety of material, etc. Analytical models for the above performances are derived and the optimized stage is fabricated from Al-7075 alloy by the wire electrical discharge machining (EDM) process for experimental demonstrations. The results provide a sound basis in developing an alternative piezo-stage for micro/nanoscale manipulation. The design and control methodology can be extended to other types of stages as well.

Index Terms—Finite-element analysis (FEA), flexure mechanisms, mechanism design, micro/nanopositioning, motion control, parallel manipulators, piezoelectric hysteresis.

I. INTRODUCTION

FLEXURE-BASED compliant micropositioning stages are the devices capable of positioning with ultrahigh precision based on elastic deformations of the structures, and they find broad applications in microelectromechanical systems (MEMS) sensors and actuators, optical fiber alignment, biological cell manipulation, and scanning probe microscopy (SPM), etc. In consideration of high-resolution requirement, the stages are usually driven by unconventional motors, such as stack piezoelectric actuators (PZTs), voice coil motors, magnetic levitation motors, and so on. A great number of compliant stages with various types of motions can be found in the literature, e.g., [1]–[10].

In particular, XYZ positioning stage is an ideal choice for some situations, where a 3-D translation is sufficient, e.g., the scanning device in an atomic force microscope (AFM). Several compliant XYZ stages are even commercially available on the market. For instance, the XYZ stage produced by the Physik Instrumente GmbH & Co. KG adopts a stacked structure of three one-degree-of-freedom (1-DOF) positioning stages. The serial connection of three stages enables a simple control strategy because the X, Y, and Z translations can be governed independently, which is at the cost of a low resonant frequency of the mechanism since the stacked stage increases the mass of moving components. In such an application as AFM, a high-speed positioning of the stage is required to implement a rapid scanning task. Thus, high resonant frequency is preferred in mechanism design of the positioning stage.

To conquer the above shortcomings of serial stages, XYZ stages with parallel-kinematics architectures [11], [12] have gained extensive attentions. Many XYZ stages with parallel kinematics have been reported in the literature. For example, several flexure hinge-based parallel stages with orthogonal structures (Delta cube) are presented in [13] and [14], a 3-DOF parallel translational stage with three identical (P+3RRR2) limbs is reported in [15], an XYZ micromanipulator based on

Manuscript received February 07, 2010; revised June 20, 2010; accepted August 14, 2010. Date of publication October 07, 2010; date of current version April 06, 2011. This paper was recommended for publication by Associate Editor S. Fatikow and Editor K. Bohringer upon evaluation of the reviewers’ comments. This work was supported in part by the Macao Science and Technology Development Fund under Grant 016/2008/A1 and the Research Committee of the University of Macau under Grant UL016/08-Y2/EME/ LYM01/FST.

The authors are with the Department of Electromechanical Engineering, Faculty of Science and Technology, University of Macau, Taipa, Macao SAR, China (e-mail: yml}@umac.mo; qxu@umac.mo).

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Digital Object Identifier 10.1109/TASE.2010.2077675


P stands for prismatic joint and R represents revolute joint.
on the Delta-type parallel robot is proposed in [16], and a novel planar parallel-kinematics stage along with out-of-plane motion is described in [17] to generate the three axial translations. However, most of the existing XYZ stages have coupled motions. In some situations where the sensory feedback for the displacements of stage output platform cannot be realized, a decoupled XYZ stage with proper calibrations is preferable [5]. Moreover, a decoupled stage will also benefit the controller design process because single-input-single-output (SISO) control scheme is sufficient.

Generally, a decoupled stage implies that one motor produces only one directional output motion without affecting the motions in other axes. The term of “decoupled” refers to the output motion decoupling of the stage. In contrast, the input decoupling [18] is rarely regarded since it emphasizes on the isolation of the input motion instead. When the stage is driven by one motor, other motors may suffer from unwanted loads due to the movement of the output platform, or clearances between other motors and the interfacing points with the stage may occur. Even though the clearances problem can be solved by using preloaded springs, the presence of unwanted transverse loads may damage some types of motors such as PZT. To cope with such difficulties, a concept of totally decoupling is presented in recent works [19] of the authors to design a compliant-based XY positioning stage with both input and output decoupling properties so as to isolate/protect actuators and obtain a decoupled output motion.

In this research, the design of an XYZ totally decoupled parallel stage (TDPS) is outlined. Among previous works on XYZ compliant parallel stages, the one presented in [20] can be classified into this category. Such an XYZ stage consists of a total of nine individual prismatic (P) hinges, which are assembled together to construct a 3-PPP parallel mechanism indeed. Intuitively, the excessive assembly of the complex joints may degrade the accuracy of the stage. Additionally, the presented stage is directly driven by linear motors, which results in a stage workspace equal to the travel stroke of the motors. In case that the stroke is not satisfied, a displacement amplification device is desirable. However, additional displacement amplifiers for this stage will complicate its structure even more. So, the predominant goal of the current research is to design and develop an XYZ TDPS with a displacement amplifier and an architecture as simple as possible for the ease of manufacture.

On the other hand, the major problem of piezo-driven stages comes from the nonlinearities due to hysteresis and creep effects introduced by PZT. The hysteresis is a nonlinear relationship between the applied voltage and output displacement and dominates the nonlinearities. Thus, once the stage is developed, the hysteresis has to be suppressed by a careful controller design in order to meet the accuracy requirement for ultrahigh precision positioning. In the literature, various approaches are proposed to compensate for the piezoelectric hysteresis effects. Specifically, both hysteresis model-based (e.g., Preisach model [21], Maxwell model [22], Prandtl-Ishlinskii model [23]–[25], and Bouc-Wen model [26]) and hysteresis model-free control schemes (e.g., inversion-based technique [27], [28], $H_{\infty}$ robust control [29], sliding mode control [30], and adaptive control [31]) have been exploited. In the current research, the hysteresis is modeled using the modified Prandtl-Ishlinskii (MPI) model [23], [25], which can better describe the nonsymmetric hysteresis. A hybrid control strategy employing the inverse hysteresis model-based feedforward combined with a proportional-integral-derivative (PID) feedback control is realized in real time to achieve a submicron accuracy of the XYZ micropositioning system.

In the rest of this paper, the mechanism design procedures of the new XYZ stage are addressed in Section II. Analytical models for the prediction of kinematics, statics, stress, and dynamics properties of the stage are established in Section III in details. The models are then validated by finite-element analysis (FEA) carried out in Section IV. Afterwards, the stage dimension is optimized in Section V and a prototype XYZ stage is fabricated in Section VI along with open-loop performance disclosed. Then, Section VII presents the hysteresis model identification and hybrid controller design processes, where extensive experiments are conducted to demonstrate the positioning capability of the developed XYZ stage. Finally, some concluding remarks are summarized in Section VIII along with future works indicated.

II. MECHANISM DESIGN PROCEDURE

The design process for a totally decoupled XYZ parallel stage is described in this section. The major objective for the design of an XYZ stage with decoupled output motion is to eliminate the cross-axis coupling errors between the X, Y, and Z directional translations and parasitic rotation errors around the axes. On the other hand, since the stage will be driven by three linear motors, it is important to ensure that when one motor drives the stage to move along the pertinent axis, the motor bears neither transverse loads nor transverse displacements induced by the other two motors driving the stage along the remaining two axes. Owing to the large output force and stiffness, PZTs are adopted to drive the stage. Besides, the hinge with right-circular shape is employed in this research as an illustration although other types of hinges can be used as well.

A typical 1-D planar translational stage employing a compound parallelogram flexure is depicted in Fig. 1(a). By applying a force $F_x$ along the $x$ axis, the output stage exhibits a 1-D translation in only one direction ($x$ axis) while without the cross-axis error in the other direction ($z$ axis). It is known that the pure translation of the output stage is enabled by assigning the same length ($l_b$) to the four legs. Alternatively, the cross-axis translation (along the $z$ axis) occurs on the secondary stage. Based on the conventional 1-D stage, a 2-D spatial translational stage without cross-axis errors is proposed in the subsequent discussions.

A. 2-D Translational Stage Without Cross-Axis Errors

By orthogonally adding a pair of flexure hinges to each leg of the conventional 1-D translational stage, a 2-D spatial stage is created, as shown in Fig. 1(b). The fixed end point in Fig. 1(a) becomes the actuation guided point in Fig. 1(b) because this point will be actuated (with a force $F_z$), as shown in Fig. 2(a) later.

It is known that a free physical body has three translational and three rotational DOFs. Some DOFs of the 2-D stage are
blocked because the corresponding motions are confined. As depicted in Fig. 1(b), the stage has four free DOFs arising from three translations along the three axes and one rotation \((\theta_z)\) around the \(x\) axis, whereas the other two rotational DOFs \((\theta_x\) and \(\theta_y)\) are constrained. By applying an external force \(F_x\), the output stage displays a pure translation along the \(x\) axis due to the same length \(l_3\) for the four legs. In addition, by applying a force \(F_y\) with an additional moment \(M_x\) to the output stage, the stage can produce a pure translation along the \(y\) axis, while without cross-axis errors in neither \(x\) axis nor \(z\) axis direction thanks to the same distance \(l_3\) assigned between the two flexure hinges in each leg. It is noticeable that in the XYZ stage designed later, the additional moment \(M_x\) for one limb mentioned above is produced by the other two limbs, i.e., the rotation \((\theta_x)\) is blocked by the other two limbs constructing the overall stage. Furthermore, due to the orthogonal nature between the rotational axes of the two sets of flexure hinges (pertinent to \(l_5\) and \(l_3\), respectively), the output stage of the proposed 2-D translational stage owns two decoupled translations in the \(x-y\) plane without cross-axis errors in the \(z\) axis, which can be easily verified by FEA.

**B. Decoupled XYZ Stage With Motor Isolation**

The phenomenon that the proposed 2-D spatial stage exhibits two translations along the \(x\) and \(y\) axes, while maintains unaffectedly in the \(z\) axis allows the conceptual design of a decoupled XYZ stage, as illustrated in Fig. 2(a), where the idea of orthogonal arrangement [32] is adopted to assemble the three limbs together. According to the screw theory, the output motion of a parallel manipulator is the intersection of the three identical limbs’ motions [20]. Thus, the DOFs of the stage can be deduced by considering free DOFs of the three limbs, as shown in Fig. 2(a)

\[
\{X\ Y\ Z\theta_y\} \cap \{X\ Y\ Z\theta_z\} \cap \{X\ Y\ Z\theta_x\} = \{X\ Y\ Z\}. \tag{1}
\]

In addition, the three limbs are assembled in such a way that the actuation axes of the limbs intersect at one common point \(P\). This assembly scheme is adopted to eliminate unwanted internal moments which will occur if the actuation axes do not intersect at one common point. However, if the motor is directly mounted between the ground and the translational stage, the end of the motor interfacing with the stage will suffer from transverse loads although it may prohibit transverse displacements with a rigid connection at the end point. The exerted transverse loads will cause damages to linear motors such as PZT. This is one of the reasons why the actuation decoupling or isolation is necessary in designing an XYZ stage.

In order to alleviate the influence of transverse loads on the linear motor, the motor has to be connected to the stage through a decoupler, whose roles are to transmit axial force of a motor and prevent the motor from bearing transverse loads [5]. It follows that the decoupler should possess both a high compliance in its working direction and a high stiffness in the transverse direction at the same time. The design of such a decoupler with large transverse stiffness by employing double compound parallelogram flexures are enumerated in [19]. Here, a decoupler as shown in Fig. 2(b) is adopted to isolate the three motors. In addition to the role of a decoupler, the employed flexure also acts as a bridge-type displacement amplifier [33], [34]. The input displacement will be amplified and transmitted as a vertical output displacement on the output end of the amplifier.

By connecting the three PZTs with 2-D stages via three decouplers/amplifiers, an XYZ TDPS is constructed, as shown in Fig. 3, where each limb is designed as a monolithic structure. For easy fabrications using such processes as wire electrical discharge machining (EDM), the amplifier and the PP limb can be machined separately instead and then assembled together.
in a two-layered manner, as depicted in Fig. 4(a). Correspondingly, the structure of the XYZ TDPS is improved, as shown in Fig. 4(b). Both input decoupling and output decoupling properties are expected from the XYZ stage. The stage performances are assessed in the following discussions by establishing analytical models first.

III. MECHANICAL MODELING

In view of the decoupling design procedure as addressed in previous discussions, it is assumed that the XYZ stage owns a totally decoupled property for the convenience of analytical modeling. Actually, this assumption is justified by the FEA carried out in Section IV later. Based on the assumption, it can be deduced that the properties of the XYZ stage in the three working axes are identical. In this section, simple analytical models of the stage for the evaluation of its amplification ratio, input stiffness, and resonant frequency are derived, which can be used for the purposes of performance assessment and cost-effective optimal design of the stage.

Fig. 5 depicts the parameters of a right-circular flexure hinge and its simplified model.

A. Kinematics and Statics Modeling

Given the input displacements ($d_1$, $d_2$, and $d_3$) of the three PZT actuators, the stage output motion ($d_x$, $d_y$, and $d_z$) and the actuation input forces ($F_1$, $F_2$, and $F_3$) can be calculated by the following kinematics and statics equations:

\[
\begin{bmatrix}
    d_x \\
    d_y \\
    d_z
\end{bmatrix} =
\begin{bmatrix}
    A_{a} & 0 & 0 \\
    0 & A_{a} & 0 \\
    0 & 0 & A_{a}
\end{bmatrix}
\begin{bmatrix}
    d_1 \\
    d_2 \\
    d_3
\end{bmatrix}
\tag{2}
\]

\[
\begin{bmatrix}
    F_1 \\
    F_2 \\
    F_3
\end{bmatrix} =
\begin{bmatrix}
    K_{a} & 0 & 0 \\
    0 & K_{a} & 0 \\
    0 & 0 & K_{a}
\end{bmatrix}
\begin{bmatrix}
    d_1 \\
    d_2 \\
    d_3
\end{bmatrix}
\tag{3}
\]

where $A_{a}$ is the amplification ratio of the displacement amplifier, and $K_{a}$ is the input (or actuation) stiffness of the stage, respectively.

Through the above relations, the kinematics and statics problems are converted to the calculations of amplification ratio and input stiffness of the XYZ stage, respectively. In the following discussions, it is assumed that the stage is driven by the PZT #2 with an input displacement $d_{in}$ and corresponding input force $F_{in}$. It produces an output displacement $d_{out}$ for the stage output platform in the $y$ axis direction. Whereas the PZTs inside limbs 1 and 3 will remain uninfluenced thanks to the roles of motor decouplers.

1) Force Analysis of the Amplifier: Owing to the double symmetric property, one quarter of the amplifier (#2) is picked out, as shown in Fig. 6(a), for the purpose of analysis. It is observed that one half of the input force, i.e., $F_{in}/2$, is applied at the input end $A$ of one quarter of the amplifier. Besides, the output end $B$ suffers from one half of the force $F_{By}$ exerted by the remaining part of the stage excluding this amplifier.

Moreover, the free body diagram of one leg $A_1B_1$ of the amplifier is sketched in Fig. 6(b). In consideration of the force and moment balance at the equilibrium state, we derive

\[
F_{A1r} = F_{B1r} = F_r
\tag{4}
\]

\[
F_{A1y} = F_{B1y} = F_y
\tag{5}
\]

\[
F_zI_2 = F_yI_1 + 2M_r
\tag{6}
\]
where $M_r$ denotes the internal moment arising from the rotation of the flexure hinge around its working axis with an angle $\Delta \alpha$, which can be expressed by

$$M_r = K_r \Delta \alpha. \tag{7}$$

Combining (6) and (7) together and considering that

$$l_1 = l_a \cos \alpha \quad \text{and} \quad l_2 = l_a \sin \alpha \quad (l_a \text{ is the length of leg } \text{A}_1 \text{B}_1),$$

yields

$$F_1 l_2 \sin \alpha - F y l_2 \cos \alpha = 2K_r \Delta \alpha. \tag{8}$$

Applying the principle of virtual work to the leg $\text{A}_1 \text{B}_1$ allows the generation of

$$F_x \Delta x - F_y \Delta y = F_1 \Delta l + 2M_r \Delta \alpha \tag{9}$$

where $\Delta x$ and $\Delta y$ are the input and output displacements of one quarter of the amplifier, and $F_1$ is the internal axial force in leg $\text{A}_1 \text{B}_1$ along the direction which causes a deformation $\Delta l$ of each flexure hinge along its longitudinal direction. In addition, the force status at the point $\text{A}_1$ is exhibited in Fig. 6(c). In consideration of the force equilibrium in the $x$ axis, we can deduce that

$$F_1 = F_x / \cos \alpha = K_f \Delta l. \tag{10}$$

Additionally, in view that $l_2$ and $\alpha$ are varying during the operation, differentiating the equation $l_2 = l_a \sin \alpha$ with respect to time allows the derivation of the relationship

$$\Delta y = l_a \cos \alpha \Delta \alpha. \tag{11}$$

2) Output Stiffness Analysis of the Stage: The force $F_y$ in (5) can be written in terms of the output displacement $\Delta y$ as follows:

$$F_y = K_B \Delta y \tag{12}$$

where $K_B$ denotes the output stiffness of the XYZ stage at the point $B$ excluding the amplifier (#2) as illustrated in Fig. 7.

Once the stage is driven by PZT #2 with an input displacement $d_{in}$, an output displacement $d_{out}$ is produced on the stage output platform, whereas the decouplers in limbs 1 and 3 will hold still since they have a large transverse stiffness. The hinge rotation angles ($\dot{\theta}$) in the two limbs are all identical due to the identical length of $l_2 = l_3$. So, the potential energy stored in these two parallel limbs can be derived as follows:

$$P_{13} = \frac{1}{2} K_{13} \dot{\theta}^2 = 16 \times \frac{1}{2} K_r \dot{\theta}^2 \tag{13}$$

where the rotation angle is $\dot{\theta} = (d_{out} / 2)/l_3$. Hence, the stiffness value $K_{13}$ can be solved by:

$$K_{13} = 4K_r / l_3^2. \tag{14}$$

Besides, the deformation of limb 2 excluding the amplifier arises from the longitudinal stiffness ($K_f$) of the 16 flexure hinges. Considering the serial/parallel connection relationships of these 16 linear springs, a necessary calculation allows the generation of the stiffness of limb #2:

$$K_2 = K_f / 4. \tag{15}$$

In view of the serial connection of $K_{13}$ and $K_2$, the output stiffness can be derived as

$$K_B = \frac{K_{13} K_2}{K_{13} + K_2} = \frac{4K_f K_r}{16K_r + \frac{l_2^2}{2} K_f}. \tag{16}$$

Moreover, taking into account the force and displacement relationship at the points $B$ and $P$ (see Fig. 7), we can obtain

$$F_{By} = K_B 2 \Delta y = K_{13} d_{out}. \tag{15}$$

3) Amplification Ratio and Input Stiffness Calculation: The following procedures show how to express the arguments $\Delta x$, $\Delta y$, $\Delta l$, and $\Delta \alpha$ in terms of $F_x$ only.

First, the variable $\Delta l$ can be derived from (10) as

$$\Delta l = \frac{F_x}{K_f \cos \alpha}. \tag{17}$$

Then, submitting (12) and (11) into (8) results in an equation of $F_y$ and $\Delta \alpha$, which further gives

$$\Delta \alpha = \frac{F_x l_2 \sin \alpha}{2K_r + \frac{l_2^2}{2} K_B \cos^2 \alpha}. \tag{18}$$

Besides, inserting (17) into (11) produces

$$\Delta y = \frac{F_x l_2^2 \cos \alpha \sin \alpha}{2K_r + \frac{l_2^2}{2} K_B \cos^2 \alpha}. \tag{19}$$

In consideration of the relationships (7), (10), and (12), the expression (9) can be rewritten as follows:

$$F_x \Delta x - K_f \Delta y^2 = K_f \Delta l^2 + 2K_r \Delta x. \tag{20}$$

Afterwards, submitting (16), (17), and (18) into (19) produces an expression of $F_x$ and $\Delta x$, which further allows the generation of

$$F_x = \frac{2K_r + \frac{l_2^2}{2} K_B \cos^2 \alpha + \frac{l_2^2}{2} K_f \cos^2 \alpha \sin^2 \alpha}{K_f \cos^2 \alpha (2K_r + \frac{l_2^2}{2} K_B \cos^2 \alpha)}. \tag{21}$$

Therefore, in view of (18) and (20), the amplification ratio of the amplifier can be obtained by

$$A_y = \frac{2 \Delta y}{2 \Delta x} = \frac{\frac{l_2^2}{2} K_f \cos^2 \alpha \sin \alpha}{2K_r + \frac{l_2^2}{2} K_B \cos^2 \alpha + \frac{l_2^2}{2} K_f \cos^2 \alpha \sin^2 \alpha}. \tag{22}$$

The input stiffness of the stage can also be derived from (20) as

$$K_a = \frac{2F_x}{2 \Delta x} = \frac{2K_f \cos^2 \alpha (2K_r + \frac{l_2^2}{2} K_B \cos^2 \alpha)}{2K_r + \frac{l_2^2}{2} K_B \cos^2 \alpha + \frac{l_2^2}{2} K_f \cos^2 \alpha \sin^2 \alpha}. \tag{23}$$

where $2F_x$ means the force applied on one quarter of the amplifier which produces a displacement of $\Delta x$, since $F_x$ is the force exerted by one leg of the amplifier.
and (b) input stiffness $K_a$ obtained of the hinges. According to the geometry of the is applied and decreasing of.

and axial loads $\sigma_t$ of flexure hinges are taken into account to derive the maximum stress, since the axial tensile or compressive stress of the flexure hinge is far less than the maximum bending stress. Thus, one has

$$\max\{\sigma_t\} \leq \sigma_a = \sigma_y / n_a$$

(24)

where $n_a > 1$ is an assigned safety factor, and $\sigma_y$ denotes the yield strength of the material.

For a flexure hinge bearing a bending moment around its rotation axis, the maximum angular displacement $\theta_{\text{max}}^\text{max}$ occurs when the maximum stress $\sigma_{r,\text{max}}$, which occurs at the outermost surface of the thinnest portion of the hinge, reaches to the yield stress $\sigma_y$. The relationship between the maximum bending stress and maximum rotational deformation of the flexure hinge has been derived in [36]

$$\sigma_{r,\text{max}} = \frac{E(1 + \beta^2/2)}{\beta f(\beta)} \rho_{\text{max}}$$

(25)

where $\beta = t/2r$ is a dimensionless geometry factor, and $f(\beta)$ is a dimensionless compliance factor defined as

$$f(\beta) = \frac{1}{2\beta + \beta^2} \left[ \frac{3 + 4\beta + 2\beta^2}{(1 + \beta)(2\beta + \beta^2)} \right] + \frac{6(1 + \beta)}{(2\beta + \beta^2)^3/2} \tan^{-1} \left( \frac{2 + \beta}{\beta} \right)^{1/2}. $$

(26)

Assume that the maximum input displacement $Q$ is applied on the input end of the amplifier in limb #1 of the XYZ stage, which results in a linear deflection $\rho_{\text{max}}^\text{max} = A_0 Q$ of the platform along the $x$ axis direction due to the maximum rotational deformation $\theta_{\text{max}}^\text{max}$ of the hinges. According to the geometry of the stage, the maximum angular deflection may occur on the hinge belong to either the amplifier in limb #1 or the compound parallelogram flexures in limbs #2 or #3.

The maximum rotation angles occurring at the three limbs can be derived as follows:

$$\theta_1^\text{max} = \frac{A_0 Q / 2}{\sqrt{l_1^2 + l_2^2}}, \quad \theta_2^\text{max} = \frac{A_0 Q / 2}{l_3}, \quad \theta_3^\text{max} = \frac{A_0 Q / 2}{l_4}.$$  

(27)
Substituting (27) into (25) allows the derivation of the relationships

\[ \sqrt{I_1^2 + I_2^2} \geq \frac{E(1 + \beta)^{9/20} \eta_a A_Q}{2 \beta^2 f(\beta) \sigma_y} \]  

\[ l_8 \geq \frac{E(1 + \beta)^{9/20} \eta_a A_Q}{2 \beta^2 f(\beta) \sigma_y} \]  

\[ l_9 \geq \frac{E(1 + \beta)^{9/20} \eta_a A_Q}{2 \beta^2 f(\beta) \sigma_y} \]  

which will provide a guideline for the design of the stage dimensions without the risk of inelastic deformations.

C. Resonant Frequency Calculation

In order to fully describe the free vibrations of the XYZ stage, the independence of the three secondary stages should be considered as well. Thus, nine generalized coordinates are selected as follows for the dynamic modeling purpose:

\[ \mathbf{q} = [d_1 \ d_2 \ d_3 \ u_1 \ u_2 \ u_3 \ u_4 \ u_5 \ u_6]^T \]  

where \( d_i \) denotes the input displacement of the \( i \)th motor, \( u_1- u_3 \) and \( u_4- u_6 \) describe two types of translations of the three secondary stages, as shown in Fig. 4(a). The parameters of one limb (\#2) are also depicted in Fig. 4(a).

The kinetic (\( T \)) and potential (\( V \)) energies for the entire stage can be expressed in terms of the generalized coordinates only. Afterwards, substituting the kinetic and potential energies into the Lagrange’s equation

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = F_i \]  

where \( F_i \) denotes the \( i \)th actuation force, allows the generation of dynamic equation describing a free motion of the stage

\[ \mathbf{M} \ddot{\mathbf{q}} + \mathbf{K} \mathbf{q} = \mathbf{0} \]  

where the mass and stiffness matrices take on the following forms shown in the equation at the bottom of page, whose matrix factors are not presented here due to the space limit.

Based on the theory of vibrations, the modal equation can be derived as:

\[ (\mathbf{K} - \omega_j^2 \mathbf{M}) \Phi_j = \mathbf{0} \]  

where the eigenvector \( \Phi_j \) (for \( j = 1, 2, \ldots, 9 \)) represents a modal shape and eigenvalue \( \omega_j^2 \) describes the corresponding natural cyclic frequency, they can be obtained by solving the characteristic equation

\[ |\mathbf{K} - \omega_j^2 \mathbf{M}| = \mathbf{0}. \]  

Then, the natural frequency can be computed as \( f_j = (1/2\pi) \omega_j \).

The lowest one among the nine natural frequencies is taken as the resonant frequency of the stage mechanism.

IV. MODEL VALIDATION WITH FEA

The established analytical models for the calculation of amplification ratio, input stiffness, and resonant frequency of the XYZ stage are verified by FEA with ANSYS software package. The architecture parameters of the stage are tabulated in Table I, where all the hinges are designed as the identical dimension. In addition, the material is assigned as Al-7075 alloy with the main parameters described in Table II.

When a force (3.6 N) is applied at the two input ends of amplifier \#2, a static structural analysis is carried out and the stage deformations along the three working direction are obtained by the FEA simulation. The corresponding input (4.15 \( \mu \text{m} \)) and predominant \( y \) axis output (15.74 \( \mu \text{m} \)) displacements of the output platform are obtained to determine the input stiffness and amplification ratio. The amplification ratio and input stiffness of the stage evaluated by FEA and analytical models [(23) and (22)] are compared in Table III. Taking the FEA results as “true” values for the amplification ratio and input stiffness of the stage, the deviations of the analytical model outputs can be derived as 25.1% and 0.35%, respectively.

Besides, it is observed that the parasitic motions of the output platform along the \( x \) and \( z \) axes are 0.017 \( \mu \text{m} \) and 0.028 \( \mu \text{m} \), respectively. These motions are caused by the input motion in amplifier \#2. Whereas they are all negligible since they only

\[ \mathbf{M} = \begin{bmatrix} M_{11} & 0 & 0 & 0 & 0 & M_{16} & 0 & M_{18} & 0 \\ 0 & M_{22} & M_{24} & 0 & 0 & 0 & 0 & M_{28} & 0 \\ 0 & 0 & M_{33} & M_{35} & 0 & M_{37} & 0 & 0 & 0 \\ 0 & M_{42} & M_{44} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{53} & M_{55} & 0 & 0 & 0 & 0 & 0 \\ M_{61} & 0 & 0 & 0 & 0 & M_{66} & 0 & 0 & 0 \\ 0 & M_{73} & 0 & 0 & 0 & M_{77} & 0 & 0 & 0 \\ M_{81} & 0 & 0 & 0 & 0 & 0 & M_{88} & 0 & 0 \\ 0 & M_{92} & 0 & 0 & 0 & 0 & 0 & M_{99} & 0 \end{bmatrix} \]  

\[ \mathbf{K} = \text{diag}\{K_{11} \ K_{22} \ K_{33} \ K_{44} \ K_{55} \ K_{66} \ K_{77} \ K_{88} \ K_{99}\} \]  

Table III: Kinematic and Dynamic Performances of an XYZ Stage

<table>
<thead>
<tr>
<th>Performance</th>
<th>Amplification ratio</th>
<th>Input stiffness (N/( \mu \text{m} ))</th>
<th>Resonant frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modeling</td>
<td>4.74</td>
<td>0.865</td>
<td>62.29</td>
</tr>
<tr>
<td>FEA</td>
<td>3.79</td>
<td>0.868</td>
<td>49.59</td>
</tr>
<tr>
<td>Deviation (%)</td>
<td>25.1</td>
<td>0.35</td>
<td>25.6</td>
</tr>
</tbody>
</table>
accounts for 0.11% and 0.18% of primary output motion of the stage, respectively. On the other hand, the induced maximum transverse motions at the input ends of amplifiers #1 and #3 are 0.013 μm and 0.009 μm, which solely equal to 0.31% and 0.22% of the input displacement, respectively. Therefore, the FEA results confirm both the input and output well-decoupling properties of the XYZ stage.

It is noticeable that although an input force of 3.6 N is used in the previous static FEA simulation, the results obtained by the FEA have linear relations. The obtained amplification ratio, input stiffness, and percentage of crosstalk will remain constant along with the changing of input force. Besides, the resonant frequency is obtained by conducting a modal analysis under ANSYS environment, which shows that the first natural frequency occurs at 49.59 Hz. In contrast, the dynamic model overestimates the stage natural frequency about 25.6%, as shown in Table III. Actually, the first three natural frequencies calculated by the dynamic model are all 62.29 Hz, whereas these frequencies predicted by the FEA are 49.59 Hz, 49.90 Hz, and 50.36 Hz, respectively. The similar natural frequency values indicate almost identical dynamic behavior of the XYZ stage in the three axes. The simulation results of the first three natural frequencies confirm the validity of the dynamic analysis model (31). Further validity will be conducted by comparing with that of experimental results in our future works.

Taking FEA results as the benchmark, one can observe that the maximum deviation of the derived model from the FEA results is up to 25% while the minimum deviation is less than 1%, which are acceptable in the early design stage. The offset mainly comes from two factors. One is attributed to the accuracy of the adopted equations for the compliance factors. The other issue lies in the neglect of compliances of the links between flexure hinges since these links are assumed to be rigid in the modeling procedure. A nonlinear modeling with the consideration of the links’ compliances will lead to more accurate analytical model results.

V. ARCHITECTURE OPTIMIZATION

Before the fabrication of the XYZ stage, it is necessary to determine its architectural parameters by taking into account its performances simultaneously. In order to improve the natural frequency of the stage, the output platform mass is reduced by removing unnecessary mass. The stroke of the three PZT (with the length of 50 mm) is assigned as 20 μm. In addition, the FEA results for the stage performances are taken as true values. Considering that the analytical models overestimate the stage performances with deviations around 20%, a compensation factor η = 0.8 is adopted in the optimization process to compensate for the derived models.

A. Optimization Problem Formulation

An insight into the expressions (22) and (23) for the input stiffness $K_a$ and amplification ratio $A_s$ reveals that these two performances rely on the involved parameters, respectively. Taking into account that the stiffness factors $K_r$ and $K_t$ are the functions of flexure hinge parameters $w$ and $t/r$, one can conclude that the stage performances are mainly dependent on the variables: $l_0$, $l_3$, $w$, $t/r$, and $α$. In view that the leg length $l_0$ and incline angle $α$ can be expressed by the other two parameters $l_1$ and $l_2$ directly, i.e., $l_0 = \sqrt{l_1^2 + l_2^2}$ and $α = \arctan(l_2/l_1)$, the main parameters that influence the stage performances become: $l_1$, $l_2$, $l_3$, $w$, and $t/r$.

As far as a material with a specific thickness ($w = 10$ mm in this research) is concerned, five parameters $(r, t, l_1, l_2, l_3)$ need to be optimized since other parameters can be determined by considering the length and width restrictions of the PZT with the addition of a proper assembling space. The amplification ratio of the stage is specified to guarantee a travel range no less than 160 μm for the output platform. The input stiffness should not exceed the minimum stiffness of the adopted PZT, i.e., $K_{PZT} = 10$ N/μm. Meanwhile, the stage should be designed with the elimination of plastic failures for the safety reason. The upper bounds for design variables are all limited so as to generate a compact stage. With the selection of natural frequency of the stage as an objective function, the optimization can be stated as follows.

- Maximize: Natural frequency ($f$).
- Variables to be optimized: $r$, $t$, $l_1$, $l_2$, and $l_3$.
- Subject to:
  
  1) Amplification ratio $ηA_s ≥ 8$.
  2) Input stiffness value $ηK_a ≤ K_{PZT}$.
  3) Free of inelastic deflections guaranteed by (28) with a safety factor $η_a = 1.5$.
  4) Parameter ranges: $3 \text{ mm} ≤ r ≤ 5 \text{ mm}$, $0.3 \text{ mm} ≤ t ≤ 2 \text{ mm}$, $2 \text{ mm} ≤ l_1 ≤ 20 \text{ mm}$, $1 \text{ mm} ≤ l_2 ≤ 4 \text{ mm}$, and $55 \text{ mm} ≤ l_3 ≤ 100 \text{ mm}$.

B. PSO Optimization and Results

The particle swarm optimization (PSO) is adopted in the current problem due to its superiority of performance over other methods such as direct search approach and genetic algorithm (GA) [37, 38]. The optimization is implemented with MATLAB, and the optimized dimensions are: $r = 3.00 \text{ mm}$, $t = 0.55 \text{ mm}$, $l_1 = 15.00 \text{ mm}$, $l_2 = 1.47 \text{ mm}$, and $l_3 = 55.00 \text{ mm}$, which will lead to an XYZ stage with $A_s = 9.4$, $K_a = 11.8$ N/μm, and resonant frequency $f = 124.5$ Hz, respectively.

To reveal the performances of the optimized XYZ stage, FEA simulations are carried out as well. In static FEA, with an input displacement 20 μm applied on the input ends of the amplifier #2, the total deformation of the stage is depicted in Fig. 9. It is obtained that the $y$-axis output displacement of the stage is 131.5 μm, and the corresponding input force is 264.0 N. Thus, the optimized stage has an amplification ratio of 6.58 with an input stiffness of 13.2 N/μm. In consideration of the nominal stiffness of the employed PZT (14–208 N/μm), the actuator can work properly for the drives of the stage. Besides, the maximum stress generated by FEA is 64.8 MPa which is far less than the yield stress (503 MPa) of the material. Thus, no inelastic deformation will occur in the stage structure.

The modal analysis demonstrates that the XYZ stage has a relatively lower resonant frequency of 78.7 Hz. It is the major disadvantage of the proposed structure, which arises from the secondary stages [see Fig. 1(b)]. The secondary stages contribute to decoupled output motion of the XYZ stage. On the
other hand, they add extra mass to the stage, which can be con-
sidered as the cost of achieving a totally decoupled property. Even so, the resonant frequency of the stage structure can be
magnified by increasing the stiffness and reducing the equiva-
lent mass of the stage. For instance, the material with a thinner
thickness can be used for fabrication and unnecessary mass of
the moving parts can be removed to achieve a resonant fre-
quency higher than 100 Hz. In the following discussions, a pro-
totype of the optimized XYZ stage is developed and experi-
ments are conducted for performance demonstration.

VI. PROTOTYPE FABRICATION AND PRELIMINARY EXPERIMENTS

In this section, a prototype of the XYZ stage is fabricated and
preliminary open-loop testing is conducted to demonstrate its
performance.

A. Experimental Setup

A prototype XYZ stage is fabricated, which is graphically
shown in Fig. 10. The three limbs of the stage are fabricated
by the wire-EDM process from Al-7075 alloy whose para-
eters are given in Table II. Concerning the actuation, three 20
μm-stroke PZTs (model PAS020 produced by Thorlabs, Inc.)
are adopted to drive the XYZ stage. A PCI-based D/A board
(PCI-6703 with 16-bit D/A converters from National Instru-
ments Corp.) is employed to produce analog voltages, which are
then amplified by three-axis voltage amplifiers to provide volt-
ages of 0–75 V for the drives of the PZTs. In order to measure
the output displacements of the moving platform, three laser
displacement sensors (Microtrak II, head model: LTC-025-02,
measuring range: 2.5 mm, from MTI Instruments, Inc.) are used.
The analog voltage outputs (bounded within ±5 V) of the three
sensor signal conditioners are read simultaneously by a personal
computer through a data acquisition (DAQ) board (PCI-6034E
with 16-bit A/D converters, from the National Instruments). The
resolution of the laser position detection system can be calcu-
lated as 0.038 μm. Additionally, in order to eliminate high-fre-
quency noises of the sensor readings, three analogy low-pass
filters with cutoff frequency of 200 Hz are used for the three axes, respectively. A sampling frequency of 2 kHz is adopted in
this research.

B. Open-Loop Performance Test Results

First, the open-loop static properties of the XYZ stage are ex-
perimentally tested. A low-frequency 0.2-Hz sinusoidal voltage
signal ranging from 0 to 8.5 V is provided by the D/A board,
which is then amplified (0–75 V) by the voltage amplifier and
used to drive the PZT #1. The three axial translations of the
XYZ stage are all recorded. With the open-loop voltage-driven
strategy, the PZT exhibits nonlinearity which is mainly at-
tributed to the hysteresis effects. Thus, the output-input
relationships of the stage are nonlinear. It is found that the
translational motions in the x, y, and z axes are 165.8, 5.4, and
6.5 μm, respectively. In view of the stroke (20 μm) of the PZT,
the amplification ratio of the stage can be determined as 8.29, which is larger
than the FEA result. The reason mainly lies in the preloading
effect of the PZT mounting. Since the PZT is inserted into the
mechanical amplifier and preloaded using an adjusting screw
shown in Fig. 10(c), the initial values for the parameters 〈see Fig. 6) are increased and decreased, respectively. Hence,
the ratio of \( \frac{I_x}{I_y} \) is greater than the nominal value. Thus, an
amplification ratio larger than the expected value is achieved as
predicted by Fig. 8(a).

Moreover, comparing to the predominant x axis motion, the
parasitic translations in the y and z axes account for 3.3% and
3.9%, respectively. The experimental results demonstrate the
low-level parasitic motions of the XYZ stage, which allows the
employment of SISO controller designs for the three axes in the
following works.
There are a lot of error sources causing the open-loop crosstalk (or cross errors) between the three axes of the XYZ stage, which include the manufacturing tolerance of the flexures, assembly error of the three pieces of stage component, mounting errors of the three displacement sensors with respect to sensor targets, and Abbe errors due to the offset distance between the measurement point and motion axis direction of the stage. The assembly precision is guaranteed by machining the flexures and fixing holes with a tolerance of ±5 μm in this research. Both the error sources and piezoelectric hysteresis affect the open-loop positioning accuracy of the micropositioning stage. In the subsequent discussions, a controller design is carried out to remedy the above shortcomings in order to realize a microscale positioning.

VII. CONTROLLER DESIGN AND EXPERIMENTAL TEST

In this section, the nonlinear hysteresis effects induced by three PZTs are compensated in order to obtain a submicron positioning accuracy for the XYZ stage. Specifically, the hysteresis is modeled using the modified Prandtl-Ishlinskii (MPI) model, and an inverse hysteresis model-based feedforward plus feedback control scheme is employed for the stage. Moreover, because the stage is well decoupled as verified above, the three axial motions can be treated independently, just like a serial stage. Hence, three SISO controllers are implemented for the \( x \), \( y \), and \( z \) axes of the micropositioning stage, respectively. For compactness, only the \( x \) axis controller design procedure is presented below.

A. Hysteresis Modeling and Identification

1) PI Model: First, the Prandtl-Ishlinskii (PI) model is reviewed shortly. As a subclass of the Preisach hysteresis model, the PI model is a superposition of elementary backlash or play operators. The backlash operator \( H_r \) is defined by

\[
x(t) = H_r[u, x_0](t) = \max \{ u(t) - r, \min \{ u(t) + r, x(t - T_s) \} \}
\]

(36)

where \( u \) is the voltage control input, \( x \) is the stage’s displacement response, \( r \) is the control input threshold value or magnitude of the backlash, and \( T_s \) is the sampling time interval. The initial condition of (36) is given by

\[
x(0) = \max \{ u(0) - r, \min \{ u(0) + r, x_0 \} \}
\]

(37)

where \( x_0 \) is usually but not necessarily initialized to zero. A generalized backlash operator can be obtained by multiplying \( H_r \) with a weight value \( w_h \), i.e.,

\[
x(t) = w_h H_r[u, x_0](t)
\]

(38)

where the weight \( w_h \) describes the gain of the backlash operator.

Using a linearly weighted superposition of multiple backlash operators with different thresholds and weights, the complex hysteresis can be modeled by

\[
x(t) = w^T_h H_r[u, x_0](t)
\]

(39)

where \( w^T_h = [w_{h0}, w_{h1}, \ldots, w_{hn}] \) is the weight vector, \( H_r[u, x_0](t) = [H_r[u, x_{00}](t), H_r[u, x_{01}](t), \ldots, H_r[u, x_{0n}](t)]^T \) with the threshold vector \( r = [r_0, r_1, \ldots, r_n]^T \) and the initial state vector \( x_0 = [x_{00}, x_{01}, \ldots, x_{0n}]^T \), for \( 0 = r_0 < r_1 < \cdots < r_n < +\infty \). The control input thresholds \( r_i \) can be chosen as equal intervals between the minimum and maximum voltage control input values of the piezo-driven stage.

2) MPI Model: Because the PI operator has the same symmetry property as the backlash operator with respect to center point of the loop formed by the operator, the model accuracy of PI operator will be reduced for many situations where the hysteresis loops are not symmetric. To overcome this restriction, a saturation operator is adopted to connect in serial with the hysteresis operator. The saturation operator is a weighted linear superposition of linear-stop or one-sided dead-zone operators given below

\[
S_d[x](t) = \begin{cases} 
\max \{ x(t) - d, 0 \}, & d > 0 \\
0, & d = 0
\end{cases} 
\]

(40)

Based on the above dead-zone operator, the saturation operator is expressed as

\[
z(t) = w^T_s S_d[x](t)
\]

(41)

where \( x \) is the output of the hysteresis operator, \( z \) is the stage response, \( w^T_s = [w_{s0}, w_{s1}, \ldots, w_{sm}] \) is the weight vector, \( S_d[x](t) = [S_d[x][0](t), S_d[x][1](t), \ldots, S_d[x][m](t)]^T \) with the threshold vector \( d = [d_0, d_1, \ldots, d_m]^T \), for \( 0 = d_0 < d_1 < \cdots < d_m < +\infty \). Hence, the modified PI model can be written as

\[
z(t) = \Gamma[u](t) = w^T_s S_d \overline{\Gamma}[u, x_0](t)
\]

(42)

3) Inverse MPI Model: In order to use the MPI model for a feedforward hysteresis compensation, the inverse model is required to express the voltage as a function of the position. The inverse MPI model is also PI-type and can be calculated by

\[
u(t) = \Gamma^{-1}[z](t) = w^T_s H^*_r \overline{W}^T \overline{S}^T_d[z], x_0^*(t)
\]

(43)

where \( w^T_s = [w_{s0}, w_{s1}, \ldots, w_{sm}] \) and \( w^T_s = [w^T_{s0}, w^T_{s1}, \ldots, w^T_{sm}] \) are the weight vectors, \( x_0^*(t) = [x_{00}^*, x_{01}^*, \ldots, x_{0n}^*]^T \) is the initial state vector, and the threshold vectors are \( r^* = [r_0^*, r_1^*, \ldots, r_n^*]^T \) and \( d^* = [d_0^*, d_1^*, \ldots, d_m^*]^T \), respectively. The inverse MPI model parameters are given in [23] and [25].

4) Model Identification: The key step to establish an MPI hysteresis model lies in weight parameters identification for matching the model output to experimental hysteresis data. Based on the experiment data, the thresholds \( r_i \) and \( d_i^* \) are assigned as

\[
r_i = \frac{i}{n + 1} \max \{ u(t) \}, \quad \text{for } i = 0, 1, \ldots, n
\]

(44)

\[
d_i^* = \frac{j}{m + 1} \max \{ x(t) \}, \quad \text{for } j = 0, 1, \ldots, m
\]

(45)
The comparisons of the experimental output and simulated model output are illustrated in Fig. 11(b). Due to the space limit, the identified MPI and inverse MPI model parameters are not presented in the paper. It is observed from Fig. 11(b) and (c) that the MPI model cannot exactly represent the nonsymmetric hysteresis of the micropositioning stage arising from the ad hoc PZT adopted in this research. A relative large difference exists between the identified model output and experimental result. The maximum model error is 5.2 μm occurring at the lower turning point at 8.5 s [see Fig. 11(b)], which accounts for 3.2% of travel range of the stage. The model error can be further reduced by adopting larger numbers of \( n \) and \( m \) while at the expense of a longer identification time for the PSO procedure. To compensate for the model errors and other uncertainties, a combined control scheme is designed below.

### B. Motion Controller Design

The purpose of motion controller design is to determine the input voltages applied to PZTs once the desired position trajectory of the XYZ stage is given. The block diagram of the control scheme is shown in Fig. 12, which indicates that a feedforward (FF) combined with feedback (FB) control strategy is employed in this research.

Based on the inverse MPI hysteresis model (43), the feedforward item can be derived by

\[
u_{FF}(t) = \Gamma^{-1} [x_d](t) = \mathbf{w}_h^T \mathbf{h}[u, x_0](t) - \mathbf{w}_s^T S_s^{\ast}[\dot{x}](t) \tag{47}\]

where \( t \) represents the time variable and \( x_d \) is the desired position trajectory.

Due to the model errors as revealed in the preceding discussions, the hysteresis effects cannot be totally compensated by the FF control (47). Therefore, a FB control is adopted to compensate for the model imperfection and other disturbances of the system. Specifically, a PID feedback controller is used due to its robustness and easy-to-implement properties. The FB control input can be written as

\[
u_{FB}(t) = K_p e_x(t) + K_i \int_0^t e_x(\tau) d\tau + K_d \frac{de_x(t)}{dt} \tag{48}\]

where the tracking error is defined as \( e_x(t) = x_d(t) - x(t) \) with \( x \) denoting the measured position. Besides, the three control parameters \( K_p, K_i, \) and \( K_d \) are proportional, integral, and derivative gains, respectively.
For the convenience of real-time digital control, the overall control signal is derived in a discrete-time form

\[ u(t_k) = u_{FF}(t_k) + K_{PF} e_x(t_k) + K_i \sum_{i=1}^{k} e_x(t_i)T_s \\
+ K_d \frac{e_x(t_k) - e_x(t_{k-1})}{T_s} \]  

(49)

where \( k \) is the index of time series and \( T_s \) represents the sampling time interval (\( T_s = 0.0005 \text{ s} \) in this research).

### C. Closed-Loop Experimental Test

1) Single-Axis Motion Tracking: First, an experiment for a staircase input signal with 0.18-\( \mu \text{m} \) height is performed, and the experimental results are shown in Fig. 13. It is evident that the positioning resolution of the XYZ stage is better than 0.18 \( \mu \text{m} \).

Second, the \( x \) axis motion tracking performance is also examined. The sinusoidal motion tracking results of a 100-\( \mu \text{m} \) peak-to-peak amplitude sine-wave signal with 0.5-Hz input rate are described in Fig. 14(a), which are produced by the feedforward (FF), PID feedback (FB), and feedforward plus feedback (FF+FB) control approaches. Besides, the position errors are plotted in Fig. 14(b), and three tracking-error indices are summarized in Fig. 14(c). The adopted error indices are peak-to-peak error (PPE) = \( \max(e_x) - \min(e_x) \), maximum absolute error (MAE) = \( \max(|e_x|) \), and root mean square error (RMSE) = \( \sqrt{\frac{1}{n} \sum_{i=1}^{n} e_x^2} \), where \( e_x = x_d - x \) represents the \( x \) axis position error and \( n \) denotes the number of data sets. The tracking error of the FF approach is mainly contributed by the model error of the identified hysteresis model, where the maximum error occurs at the lower turning points as indicated in the previous Fig. 11(b).

The effectiveness of the combined FF+FB control over the two separate methods is clear to observe. With the FF+FB scheme, the performances of PPE = 0.96 \( \mu \text{m} \), MAE = 0.49 \( \mu \text{m} \), and RMSE = 0.16 \( \mu \text{m} \) are achieved, which have been improved by 6.5, 10.8, and 17.1 times with comparison to those of FF control results, and enhanced by 1.9, 1.9, and 3.8 times in comparison to the standalone FB outputs, respectively.

2) Triaxial Motion Tracking: In order to discover the triaxial cooperative 3-D tracking performance of the XYZ stage, spatial contouring tests are executed. Specifically, a spherical curve contouring is performed with different feed-rate for a spherical surface of 10-\( \mu \text{m} \) radius [see Fig. 15(a)]. As for this spatial curve, the feed-rate is varying during the contouring task and the maximum feed-rate (\( V_{\text{max}} \)) arrives at the midpoint of the curve.

In the experiments, the stage output platform moves from the home position (0, 0, 0) \( \mu \text{m} \) to the starting point (10, 10, 0) \( \mu \text{m} \) in the \( x - y \) plane through a linear contouring, and then tracks the 3-D spherical curve from bottom to up. With \( V_{\text{max}} = 45 \mu \text{m/s} \), the experimental results are illustrated in Fig. 15, where Fig. 15(a) shows the contouring result, and
Fig. 15. Triaxial spherical curve contouring results for a 10-μm radius sphere with the maximum feed-rate of 45 μm/s. (a) 3-D view. (b) Three axes tracking results. (c) Tracking errors.

Fig. 15(b) and (c) depict the tracking results and errors of the three axes, respectively. It is derived that the PPE, MAE, and RMSE of all the three axes tracking errors are suppressed within 1.07 μm, 0.58 μm, and 0.10 μm, respectively. Actually, the contouring result between 0 to 1 s also reveals a well 2-D linear contouring performance of the stage with a feed-rate around 14 μm/s. Moreover, as the increasing of \( V_{\text{max}} \), the spherical contour is also tested and the tracking performances are summarized in Fig. 16. It can be observed that the tracking errors increase as the rising of the feed-rate due to the bandwidth limit of the control system.

D. Discussions on Stage Performances

It is noticeable that in the 1-D positioning experiment with results shown in Fig. 13, the PID control parameters of \( K_p = 0.006, K_i = 2.5, \) and \( K_d = 0.00001 \) are used. Whereas for the 1-D sinusoidal motion tracking, 2-D linear and 3-D spherical curve contouring, the PID gains are set as \( K_p = 0.006, K_i = 5.0, \) and \( K_d = 0.00001 \), which are tuned by trial and error during the experiments. In order to generate better control results, different sets of control gains are required for different types of input signals. This reflects one of the limitations of the conventional PID control and leaves a room for further improvement on controller design in our future research.

Moreover, it is found that the resolution of displacement sensor has a major influence on the control results. The above closed-loop experiments demonstrate that a submicron positioning accuracy of the XYZ stage can be achieved with displacement sensors of 0.038-μm resolution, which can be employed in micropositioning applications. Adopting displacement sensors with a subnanometer-level resolution, a nanometer positioning accuracy of the stage is expected for nanopositioning purposes even with the same control algorithm proposed in this research. Besides, experiments such as manipulating microscopic objects will be performed to demonstrate the ability of the XYZ micropositioning stage in the next step research.

It is noticeable that the control scheme presented in this research does not rely on the dynamic model of the piezo-driven XYZ stage. The established dynamic model (31) is only used for the resonant frequency calculation and architecture optimization of the XYZ stage. In our future work, a dynamic model-based controller will be constructed to achieve a high-bandwidth control of the micropositioning system. Robust control strategy will be implemented to suppress hysteresis, vibration, and residual modes which may cause control spillover phenomena of the system.

VIII. CONCLUSION

This paper presents the design and control of a new decoupled piezo-driven XYZ parallel micropositioning stage. The stage owns a simple structure and has both input and output decoupling properties in virtue of motor isolation and decoupled output motion. A series of analytical, simulation,
and experimental studies are undertaken to facilitate the mechanical modeling, dimension optimization, hysteresis model identification, and control experiments. The mechanism of the XYZ stage is modeled based on the flexure joint simplification, and the conducted FEA simulations verify the efficiency of the derived models for stage performance assessment. It is found that the stage has almost identical dynamic behavior in the three working axes. The open-loop experiment reveals a well-decoupling property of the stage. Moreover, closed-loop experimental studies show that a submicron accuracy single-axis motion tracking and triaxial spatial contouring has been achieved by the micro positioning stage, which validates the effectiveness of the proposed control scheme employing the inverse MPI hysteresis model-based feedforward combined with PID feedback control.

Further improvement of the research can be made on: 1) establish more accurate analytical models for the stage performance prediction; 2) design more sophisticated controller for the plant nonlinearity and uncertainty alleviation; 3) use (sub)nanometer-level resolution displacement sensors and apply the stage to micro/nanoscale objects manipulation; and 4) miniaturize the stage into micro or meso-scale for pertinent micro/nanomanipulation applications.

REFERENCES

Yangmin Li (M’98-SM’04) received the B.S. and M.S. degrees from Jilin University, Changchun, China, in 1985 and 1988, respectively, and the Ph.D. degree from Tianjin University, Tianjin, China, in 1994, all in mechanical engineering.

He is currently a Professor of Electromechanical Engineering at the University of Macau, where he also directs the Mechatronics Laboratory. He has authored about 225 scientific papers, and has served 85 international conference program committees. His research interests include micro/nanomanipulation, nanorobotics, micromanipulator, mobile and modular robot, multibody dynamics and control.

Dr. Li is a member of the American Society of Mechanical Engineers (ASME). He currently serves as Technical Editor of the IEEE/ASME TRANSACTIONS ON MECHATRONICS, Associate Editor of the IEEE TRANSACTIONS ON AUTOMATION SCIENCE ENGINEERING, a council member and an Editor of the Chinese Journal of Mechanical Engineering, and a member of editorial board of the International Journal of Control, Automation, and Systems.

Qingsong Xu (M’09) received the B.S. degree in mechatronics engineering (Hons) from Beijing Institute of Technology, Beijing, China, in 2002, and the M.S. and Ph.D. degrees in electromechanical engineering from the University of Macau, Macao SAR, China, in 2004 and 2008, respectively.

He is currently an Assistant Professor of Electromechanical Engineering at the University of Macau. His current research interests include design, analysis, and control of parallel manipulators and micro/nanomanipulators for micro/nanomanipulation, computational intelligence, advanced and intelligent control of smart actuators, and automation systems.